Construction of dual bases
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Abstract
Let $B_n := \{b_0, b_1, \ldots, b_n\}$ ($n = 0, 1, \ldots, N; \ N \in \mathbb{N}$) be the sets of linearly independent functions. We give a simple method of construction the dual functions $D_n := \{d_0^{(n)}, d_1^{(n)}, \ldots, d_n^{(n)}\}$ ($0 \leq n \leq N$) satisfying the following conditions: $\text{span} D_n = \text{span} B_n$ and $\langle b_i, d_j^{(n)} \rangle = \delta_{ij}$ ($0 \leq i, j \leq n \leq N$), where $\delta_{ii} = 1, \delta_{ij} = 0$ for $i \neq j$, and $\langle \cdot, \cdot \rangle$ is a given inner product. The proposed algorithm allows us to construct all the sets of the dual functions $D_0, D_1, \ldots, D_N$ in the time $O(N^3)$, where $N$ is a natural number. Four illustrative examples presenting the possible applications of obtained results are given.

Keywords: Dual bases; Approximation; Bernstein basis polynomials; B-spline functions.

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¹Supported by Narodowe Centrum Nauki (Poland) under the grant 2011/01/B/ST1/01221