Theorem Proving (List 5)

Deadline: 20.04.2016

1. Using DPLL with learning, decide which of the following clause sets are satisfiable. If the clause set is satisfiable, then find a model. If the clause is not satisfiable, then find a resolution refutation.

   (a) $\{A, B\}$, $\{\neg A, B\}$, $\{\neg B\}$.

   (b) $\{A, B\}$, $\{\neg A, B\}$, $\{A, \neg B\}$, $\{\neg A, \neg B\}$.

   (c) $\{\neg A, B\}$, $\{A, C, B\}$, $\{B, C\}$, $\{B, \neg C\}$.

   (d) $\{A, B, E\}$, $\{\neg A, C\}$, $\{\neg B, D, E\}$, $\{\neg A, \neg C\}$, $\{\neg B, \neg D\}, \{\neg E\}$.

   (e) $\{P, Q\}$, $\{Q, R\}$, $\{P, R\}$, $\{\neg P, \neg Q\}$, $\{\neg R, \neg Q\}$, $\{\neg R, \neg P\}$.

2. Consider clause set

   $$
   \begin{array}{cl}
   (1) & \{A\} \\
   (2) & \{\neg A, B\} \\
   (3) & \{\neg A, \neg B, C\} \\
   (4) & \{\neg A, \neg D, E\} \\
   (5) & \{\neg B, \neg D, F\} \\
   (6) & \{\neg D, \neg E, \neg F, G\} \\
   (7) & \{\neg A, \neg B, \neg G, X\} \\
   (8) & \{\neg G, Y\} \\
   (9) & \{\neg C, \neg X, \neg Y\}
   \end{array}
   $$

   In the beginning, clauses 1,2,3 are productive. After that, assume DPLL decides $D := \mathbf{t}$. The resulting state has $S =$

   $$
   \begin{array}{l}
   \mathbf{det}(\text{A, 1}) \\
   \mathbf{det}(\text{B, 2}) \\
   \mathbf{det}(\text{C, 3}) \\
   \mathbf{decide}(\text{D})
   \end{array}
   $$

   and $I(A) = I(B) = I(C) = I(D) = \mathbf{t}$.

   Continue DPLL until a conflict state is reached:

   (a) Give the state of the stack, and the interpretation in the final (conflict) state.

(b) Backtrack and learn a clause, assuming that learning uses the last decision variable.

(c) Backtrack and learn a clause, assuming that learning uses the last (nearest to the conflict) UIP.

(d) In both cases, continue DPLL until an interpretation is found.