Exercises due on Friday 23d of November 11.15

November 16, 2001

1. Expand the following formulae, inserting the $\cdots$'s that have been omitted:
   (a) $(f\ x)$,
   (b) $(f\ a\ b\ c)$,
   (c) $(f\ (g\ a\ b)\ (g\ b\ a)\ )$,
   (d) $(s\ (s\ (s\ 0)))$.

   Expand the following formulae, using Π instead of $\Rightarrow$:
   (a) $X \Rightarrow Y \Rightarrow Z$,
   (b) (List Nat) $\Rightarrow$ (List Nat) $\Rightarrow$ (List Nat).
   (also expand the $\cdot$)

2. Which of the following pairs of terms are $\alpha, \beta, \delta, \eta$-equivalent?
   (a) $\lambda x{:}X(p\ x)$ and $\lambda y{:}X(q\ x)$,
   (b) $\lambda p{:}(X \Rightarrow \text{Form})\ \lambda x{:}X\ (p\ x)$ and $\lambda q{:}(X \Rightarrow \text{Form})\ \lambda y{:}X\ (q\ y)$,
   (c) $\lambda x{:}X\ (f\ x)$ and $(\lambda y{:}(X \Rightarrow X)\ y) \cdot f$,
   (d)

   $$\text{true}$$

   and

   $$(K\ (K\ \text{true})) \cdot \text{false},$$

   in the context

   $$\text{Bool}{:}\text{Type},\ \text{true}{:}\text{Bool},\ \text{false}{:}\text{Bool},$$

   $$K := \lambda x{:}\text{Bool}\ \lambda y{:}\text{Bool}\ x{:}(\text{Bool} \Rightarrow \text{Bool}).$$

3. Derive the following type judgments, using the typing rules presented in the slides:
   (a) $\text{Nat}{:}\text{Type},\ 0{:}\text{Nat},\ s{:}\text{Nat} \Rightarrow \text{Nat} \vdash (s\ (s\ (s\ 0))){:}\text{Nat}$,
   (b) $\text{Nat}{:}\text{Type},\ 0{:}\text{Nat},\ s{:}\text{Nat} \Rightarrow \text{Nat},\ \vdash \lambda x{:}\text{Nat}\ (s\ x){:}(\text{Nat} \Rightarrow \text{Nat})$,
   (c) Bool:Type, true:Bool, nil:ΠX:Type (List X),
   cons:(ΠX:Type X $\Rightarrow$ (List X) $\Rightarrow$ (List X)) $\vdash$
   (cons Bool true (cons Bool false (nil Bool))):(List Bool).