Introduction to Formal Logic: Exercises on Semantics of Predicate Logic

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1. Run the proof search algorithm **ModSearch** on the following formulas:
   (a) $\forall x \exists y \; [\; R(x,y) \wedge x \not\approx y \;]$. (In case the formula has no constants, one needs to invent a first constant)
   (b) $\exists x \; [\; D(x) \rightarrow \forall y \; D(y) \;]$, (the Drinker paradox)

2. Let F_1, F_2 be formulas. Let A be an n-place predicate, that does not occur in F_1, F_2.

   Show that F_1 has a model iff $F_1 \wedge \forall x_1 \cdots x_n \; [\; A(x_1, \ldots, x_n) \leftrightarrow F_2 \;]$ has a model.

3. Let F be a formula, let t be a term, let f be an n-place function symbol that does not occur in F, t. Show that F has a model iff

   $$F \wedge \forall x_1 \cdots x_n \; [f(x_1, \ldots, x_n) = t]$$

   has a model.

4. Let the Post Correspondence Problem (PCP) be defined as follows:

   Let V be a sequence of words $v_1, \ldots, v_n$. Let W be a sequence of words $w_1, \ldots, w_n$.

   The problem is: Find a sequence of integers $i_1, \ldots, i_p$, where $p > 0$, each $1 \leq i_j \leq n$, and

   $$v_{i_1} \cdot v_{i_2} \cdot \ldots \cdot v_{i_p} = w_{i_1} \cdot w_{i_2} \cdot \ldots \cdot w_{i_p}.$$

   The PCP problem is known to be undecidable.

   Give a method of translating an arbitrary instance π of PCP into a first-order formula F, s.t. F has a model iff π has no solution.

   It follows that the problem 'Does formula F have a proof' is undecidable. (But it is enumerable, explain why)

5. Give a method of translating an arbitrary instance π of PCP into a first-order formula F, s.t. F has a *finite* model iff π has a solution.

   It follows that the problem: 'Does formula F have a finite model' is undecidable. (Boris Trakhtenbrot, 1950)

   (But it is enumerable, explain why)