# Introduction to Formal Logic: Exercises on Sequent Calculus for Propositional Logic

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1. Prove, using sequent calculus, the following formulas:
   (a) $\neg(A \wedge B) \rightarrow (\neg A \vee \neg B)$,
   (b) $(\neg A \vee \neg B) \rightarrow \neg(A \wedge B)$,
   (c) $A \leftrightarrow \neg\neg A$,
   (d) $(A \rightarrow B) \rightarrow (B \rightarrow C) \rightarrow (A \rightarrow C)$,
   (e) $((A \wedge A) \rightarrow B) \rightarrow (A \rightarrow B)$,
   (f) $(A \leftrightarrow B) \leftrightarrow (\neg A \leftrightarrow \neg B)$
2. Some truths involving implication can be pretty perverted. We prove them anyway.
   (a) $\neg A \rightarrow (A \rightarrow B)$,
   (b) $(A \rightarrow B) \vee (B \rightarrow A)$,
   (c) $((A \rightarrow B) \rightarrow A) \rightarrow A$, (Peirce's law)
   (d) $(A \wedge \neg A) \rightarrow B$. (reasoning by contradiction)

   The problem is that, according to our feeling $\rightarrow$ expresses some kind of law-like behaviour. In the truth table definition of $\rightarrow$ all law-like behaviour has disappeared.
3. Try to prove, using sequent calculus, the following formulas. If they are not provable, then give a counter interpretation.
   (a) $A \wedge B \rightarrow A \vee B$.
   (b) $A \vee B \rightarrow A \wedge B$.
   (c) $A \wedge (A \leftrightarrow B) \rightarrow B$.
   (d) $A \rightarrow \neg A$.
   (e) $((A \vee (A \rightarrow B)) \rightarrow B) \rightarrow B$.