A quest for new computer logics

Research project objectives/ Research hypothesis

Since the works of Church and Turing it is known that *the satisfiability problem* for first-order logic, FO, is undecidable, i.e., there is no algorithm which for a given FO-sentence checks whether this sentence has a model. This result was an inspiration for a large research programme of classifying interesting syntactic fragments of first-order logic with respect to the decidability and the computational complexity of their satisfiability problem. The decidability of fragments of first-order logic was initially investigated mostly by mathematicians and logicians. Later, however, this kind of research received strong motivations from the world of computer science, when it turned out that it has numerous connections to its various fields, including hardware and software verification, artificial intelligence, distributed computing and databases. Typical objects arising in practical applications such as data or knowledge bases, computer programs and computations can be modelled as relational structures and their properties can be specified by means of logical formulas. Having data described/specified within a logical formalism one often wants such a specification to undergo an automated analysis. Very often such an analysis task boils down just to verifying *satisfiability* or *finite satisfiability* of a logical formula.

The main goal of this project is to investigate the decidability and the computational complexity of the satisfiability problem and some fundamental related problems motivated by practical applications of some extensions of the following fragments of FO: uniform one-dimensional fragment, UF_1, the two-variable logic, FO^2, the unary negation fragment, UF_1, and the fragments with guarded negation, GNF, and guarded quantification, GF. Among the fundamental extensions of the above basic formalisms are equivalence and transitive relations, linear orders and counting quantifiers, suitable for various application purposes. We also plan to examine satisfiability of the mentioned logics in some natural classes of structures, such as words, trees and forests. Among the mentioned formalisms, FO^2 and GF are pretty well understood, but there are still some interesting open questions that we plan to answer. The main tasks of this project will however concern the logics proposed in the recent few years: UNF, GNF and UF_1.

The main objectives are (i) investigating the key problems concerning some extensions of our base languages, motivated by practical applications, (ii) designing optimal algorithms for decidable problems, (iii) examining the finite versions of the problems, (iv) analysing the expressive power of the obtained formalisms, (v) investigating some succinctness issues.

Research project methodology

We plan to apply various combinatorial, algorithmic and model-theoretic methods, which proved useful in the case of other problems from the considered field, as well as to devise some new techniques. We hope that in the new areas such as UNF, GNF i UF_1 we will be able to benefit from the experience gained during our works on modal and description logics, as well as on the classical logics with two variables and guarded quantifiers. Among the methods, whose usefulness we hope for, worth mentioning are model surgery (including tree-like unravellings and small models/substructures theorems), linear and integer programming, automata and vector addition systems, alternating Turing machines and tiling systems. We expect that some problems will require inventing completely new tools.