A Family of Approximation Algorithms for the Maximum Duo-Preservation String Mapping Problem

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Minimum Common String Partition

Input: two strings X, Y, where Y is a permutation of X.
Output: partition of X into the least number of pieces that can be rearranged (without reversing) and concatenated to obtain Y.

X : x y z a b c b x y

Y : a b b c x y z x y

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Hardness of MCSP

Goldstein, Kolman, Zheng ['04]

MCSP is APX-hard.

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Almost linear-time $O(\log n \cdot \log^* n)$-approximation algorithm.

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Maximum Duo-Preservation String Mapping Problem

Complementary problem: maximize number of **duos** – consecutive letters not split apart.

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$$|X| = \#\mathit{duos} + \#\mathit{pieces}$$

Easier? No:

Boria, Kurpisz, Leppänen, Mastrolilli ['14]:

MPSM is APX-hard.

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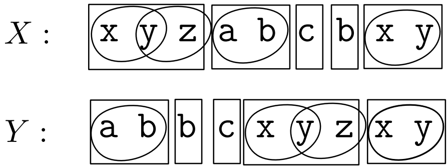

Preserved duos: xy, yz, ab, xy

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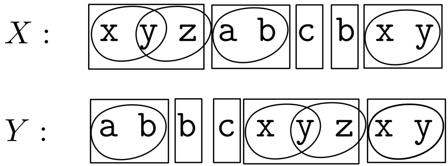

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Results for MPSM

Authors	year	ratio
Boria et al.	'14	4
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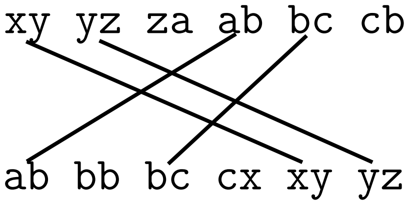
Graph representation

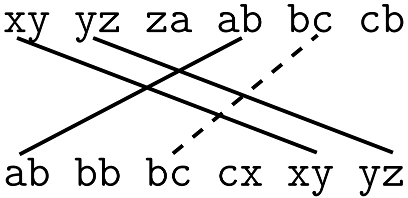
Bipartite graph with nodes – duos in both strings:

x y z a b c b

a b b c x y z

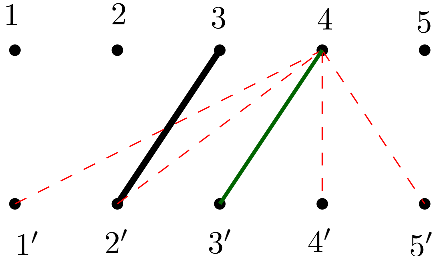


Maximum consecutive bipartite matching:



Maximum Consecutive Bipartite Matching

When we take the edge $(2', 3)$ to the matching:



Definitions

Streak:

Conflicting edges:

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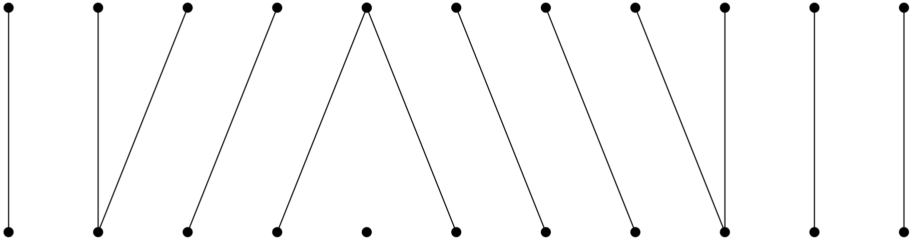
Greedy Algorithm

As long as possible take the longest possible streak from G.



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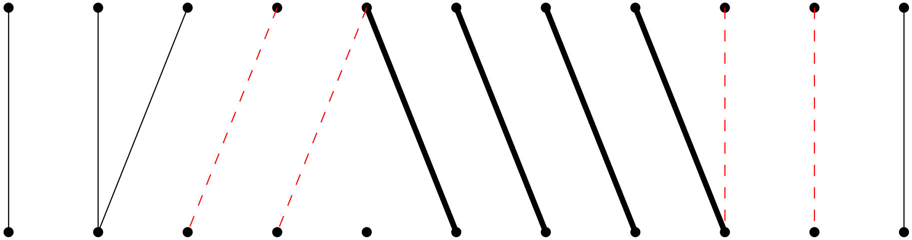

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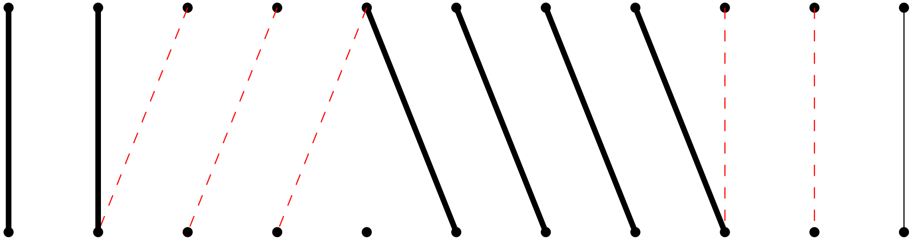

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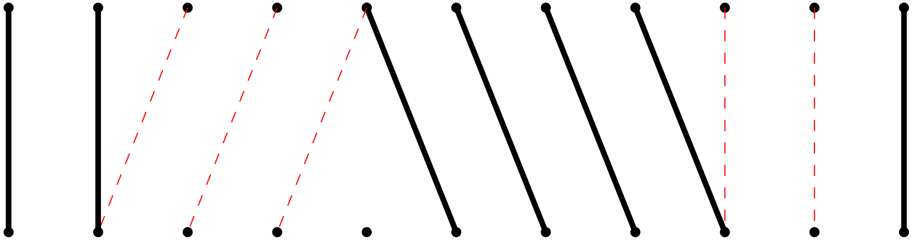

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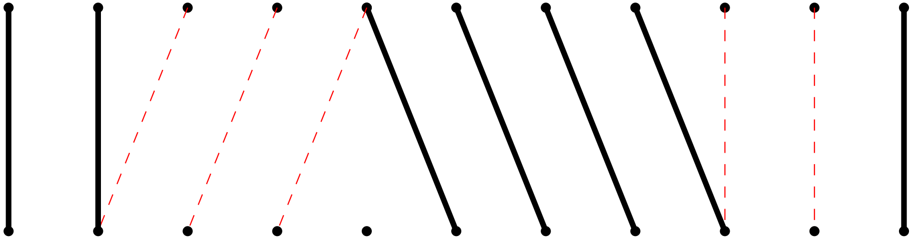

If we stop the algorithm when streaks contain less than k edges:

Lemma

There are $(2 + \frac{2}{k}) \cdot |\textsc{Greedy}|$ edges from optimal solution conflicting with $\textsc{Greedy}$.

Possible approaches for small k

Lemma

Streaks of length at least k: $(2 + \frac{2}{k})$-approximation.

Streaks smaller than k need another phase:

- $k = 1$ The greedy algorithm alone yields 4-approximation.
- $k = 2$ Maximum matching for the remaining edges yields 3-approximation.
- $k = 3$ Local search technique used by Boria et al. yields 2.67-approximation.

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$(2+\varepsilon)$-approximation

BoundedSizeImprovements(t)

- try every subset E_{add}, E_{del} of at most t edges
- if $|E_{del}| < |E_{add}|$, try $ALG \setminus E_{del} \cup E_{add}$

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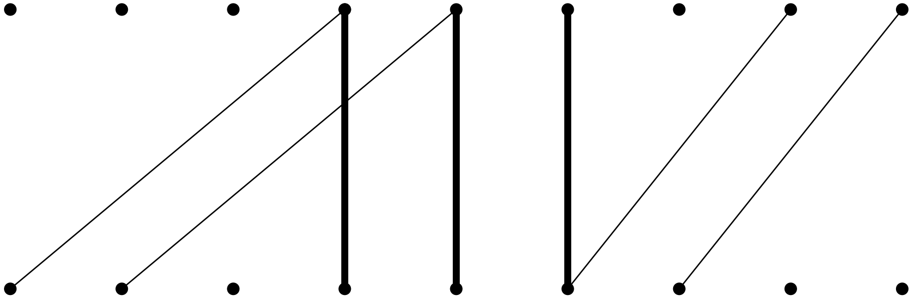

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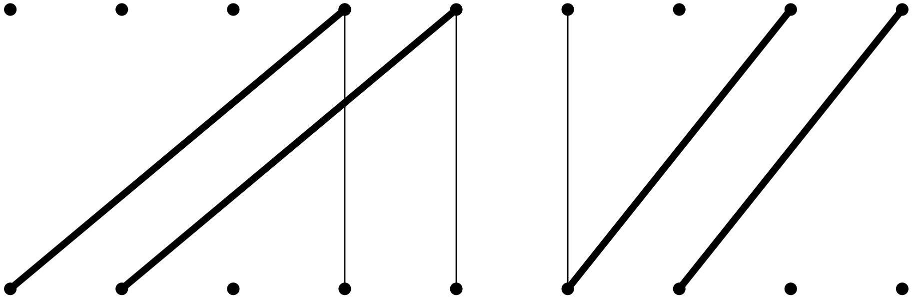

Final algorithm

1. run GREEDY for $k = \lceil \frac{2}{\varepsilon} \rceil$
2. run BOUNDEDSIZEIMPROVEMENTS($\lceil \frac{4}{\varepsilon} \rceil + 1$).

Theorem

Combining the greedy algorithm with local improvements yields a $(2 + \varepsilon)$-approximation for MCBM in $n^{O(1/\varepsilon)}$ time, for any $\varepsilon > 0$.

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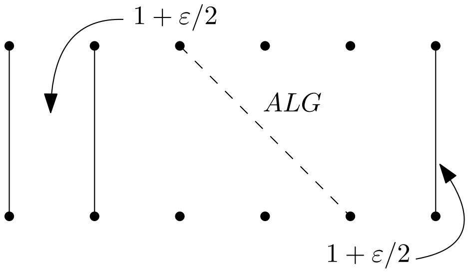
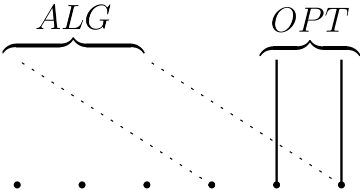
Proof: assign $2+\varepsilon$ credits to every edge from ALG



Credit distribution scheme

Often it is clear how to distribute the credit:



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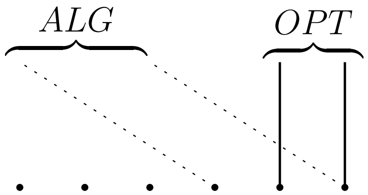

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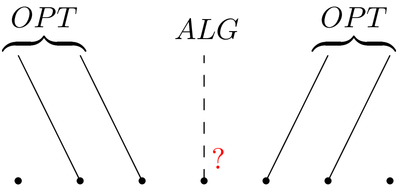


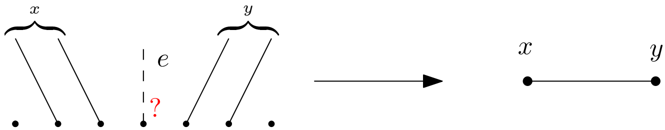
Balance of a streak

Balance: number of credits distributed to a streak minus its size.

Lemma

Balance of every streak is at least -2.

New graph with nodes – streaks of *OPT*:



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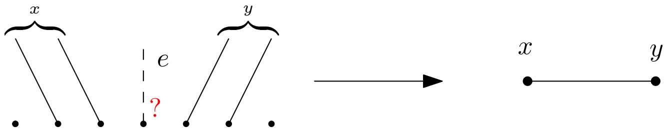

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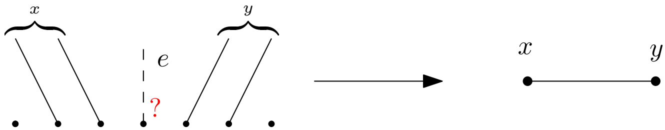

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   - upper bound: $2 + \varepsilon$
2. Is k-MPSM significantly easier than MPSM?

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