Guy Kortsarz
professor
Department of Computer Science
Rutgers University-Camden

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Review of Jan Marcinkowski's Ph. D. thesis "Three small discoveries in the field of (in-)approximability.".

The thesis first introduces computational complexity conjectures:

Small set expansion I tried for a month to disprove the small set expansion conjecture (SSEC). At the end I understood why it is probably hard. I wasted a month. This conjecture implies the unique game conjecture hence I can say that these conjectures seem solid to me.

Label Cover The Labelcover problem defined on page 1 perhaps should have another name. I used this new problem and tried to tell my co-authors not to call it Labelcover. They refused by the way. Labelcover is about choosing a label for every question and maximizing the number of queries accepted. The "new" Labelcover has a completely different (and useful, that is true) objective function. You look on how many vertices of one side are completely satisfied. See the definition of weakly satisfied which is used here. Non of that has to do with the Labelcover problem as defined originally.

Unique game conjecture The unique game conjecture (UGC) and strong unique game conjecture (SUGC) are defined as usual.

In the following, Jan Marcinkowski shows his results

Minimum Maximal Matching problem The hardness for minimum size maximal independent set is strong: better than 2 approximation is not possible under the UGC.

A less strong hardness of 4/3 is given for bipartite graphs. (To get a lower bound of 2 you need the stronger version of UGC with expansion properties for a no instance).

It is interesting that under the small set expansion conjecture a lower bound of 2 can be derived. This conjecture is stronger than the UGC, which is a less desirable. Still I think the conjecture is true.

The strategy adopted is using the fact that for an independent set you cannot tell between a set with 1/2 of the vertices in a "yes" instance and in a "no" instance the largest independent set has size at most ϵn (the epsilon must be small enough I think). This is well known result.

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The issue is that the complement of a maximal matching is an independent set. Usually, this does not help. But it helps with the parameters of the latter theorem, because the "yes" instance has a very large independent set. This presents a big problem for bipartite graph (the independent set problem is solvable in polynomial time on bipartite graphs).

Here the authors use deep results of Bansal and Khot, which is a rather difficult to read (I personally started but did not finish): We have a many functions f (who can be seen as error correcting code) so that the function is the value of one entry (or its negation). These are called *dictatorship functions*. It is possible to tell if a function is a dictatorship function (for "yes" instances) asking few bits and in a no instance *I think* it is far from being a dictatorship. I did not completely understood this result. Sorry.

The tests are very clever and hard to understand, at least the proof. The authors at least show this theorem as a black box (avoiding Fourier analysis needed for a "no" instance). The result presented is based on the above paper. The authors add edges to the construction. To create a matching between the vertices not in the independent set in a yes instance. The result of Bansal and Khot is not enough. It is interesting that what does not work is the soundness: After the edges are added the "no" instance is not influenced, but there is a perfect matching on the complement of the independent set for a "yes" instance.

The reduction for the bipartite case duplicates the vertices to two sets and connects two vertices if they were connected in the original graph. This part is easy, in fact. Then there is a reduction from the SUGC that gives a lower bound of 2. They use a reduction from SUGC to a problem of finding in a general graph a Balanced complete bipartite clique (or in the case of this paper, a Balanced independent set). This is a well known problem for which no NP-hardness is known. I understood most details here.

There is also a lower bound from SSEC. Both SSEC and SUGC are stronger assumptions than UGC. I am not aware that a relation between the two is known. Hence both hardness resuls are independent. The reduction from SSEC is based on a paper of Manuransgsi et al.

The lower bound here are impressive in my eyes and are a strong first half of a good thesis.

Approval voting The approval voting elections issue I did not understand part of it. I did not understand especially why the distribution chosen was the one chosen.

Parametrized k-Median The result on parametrized k-Median (with capacities) seems interesting to me but I have not much to say about it.

Evaluation

I can say that the Ph. D. thesis should clearly be accepted. The work here deals with some highly central problems. For example capacitated k-median, which is maybe the only

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problems we do not know anything about. Also minimum maximal vertex cover. I did not know that there was no hardness of 2 (I assumed there was in fact).

The papers in question use the most recent tools in lower bound. For example Unique game with slight expansion in a no instance, an assumption I used myself. I did not introduce it, but I was wondering if this would be accepted as it is a stronger assumption than UGC. They accepted this hardness assumption without any problem. Good to know. The student uses something similar to this paper of mine in section 5 (which is much easier for me to understand).

Jan Marcinkowski's co-authors are among the absolute best in the world in these topics. Namely, lower bounds based on the small set expansion problem and the Unique game conjecture.

For example Pasin Manurangsi may be the topmost expert on this. He gave in a paper hardness of 2 for the k-Cut problem and other famous problems like the balanced Complete Bipartite graph problem (see later). He has a rather amazing hardness based on the ETH for a famous problem: The densest k-subgraph problem. The hardness is rather amazing compared to what was known before: $n^{1/\log\log n}$.

In fact I also know how smart he is because I have a paper with him (on parametrized hardness). We did hardness for clique but Bundit said that for Set Cover it is better to ask to people from Berkely, among them Pasin. Indeed they prove hardness for Set Cover.

Everything about this thesis shows impeccable knowledge in very hard results and very recent results. Not every day I meet a student, even a Ph. D. student, who read all these difficult results. Jan Marcinkowski is clearly a strong student.

Comments

You should have removed the "small" from the heading of the thesis. Who knows what is small? The results do not seem small to me.

The remark about any life form having the same computation power is amusing but most probably not at all true. Aliens can be made out of silicon for all we know. But in our world, all assumptions seem to have at least some evidence.

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