## Probability \& Statistics

## Problem set №3. Week starting March $16^{\text {th }}$

1. $A$ and $B$ are events such that $P(A \cap B)=1 / 4, P\left(A^{C}\right)=1 / 3, P(B)=1 / 2$. Find $P(A \cup B)$.
2. Is it true that 13. day of the month is connected with Friday? (January 1, 1601 - December 31, 2000)

Explanation: Year $n$ is a leap year if $n \equiv_{4} 0$, with the exception of years divisible by $100\left(n \not \equiv_{100} 0\right)$; unless $n \equiv{ }_{400} 0$ (i.e. year 2000). How many times in 400 -year cycle 13 . day of the month was Monday, Tuesday, ..., Sunday?
Random variables $X, Y$ are independent, iff, in discrete case, condition $P\left(X=x_{i}, Y=y_{k}\right)=P\left(X=x_{i}\right)$. $P\left(Y=y_{k}\right)$ holds.
3. R.v. $X$ has binomial distribution $B\left(n_{1}, p\right)$ and r.v. $Y B\left(n_{2}, p\right)$ distribution. $X, Y$ are independent. Prove that $Z=X+Y$ has $B\left(n_{1}+n_{2}, p\right)$ distribution.
4. Independent r.vs. $X, Y$ have Poisson distribution with parameters $\lambda_{1}$ i $\lambda_{2}$. Prove that r.v. $Z=X+Y$ has Poisson distribution with parameter $\lambda_{1}+\lambda_{2}$.

Density of r.v. $(X, Y)$ has form $f(x)=3 x y$ on area bounded by $y=0, y=x, y=2-x$.
5. Find marginal densities $f_{1}(x), f_{2}(y)$.
6. Calculate expected value of $Y$. Check if r.v. $X, Y$ are independent.
7. Probability of success in independent trials equals $p$. We perform trials until 3 successes occur. R.v. $X$ is equal to number of performed trials. Find distribution of $X$, i.e. find density function (probabilities) and expected value $X$.
8. Readable and thoroughly - without using the notes - write upper and lower Greek letters: alpha $\alpha$, beta $\beta$, zeta $\zeta$, eta $\eta$, lambda $\lambda$, chi $\chi$, xi $\xi$, phi $\phi$, rho $\rho$.
9. (a) Let $X \sim U[-2,2]$. Find distribution of $Y=|X|$.
(b) Given $X \sim U[-1,1]$ find distributions of $Y=X^{3}, Z=X^{2}$.
10. Let $X$ be r.v. with geometric distribution $(X \sim \operatorname{Geom}(p))$. Check that $\mathrm{V}(X)=\frac{1-p}{p^{2}}$.
11. Cardinality of sets $A_{1}, \ldots, A_{4}$ is equal - respectively $-40,32,20,50$. An element (from set of 142 elements) is randomly chosen. Cardinality of the set from which chosen element was taken is the value of random variable $X$. Next a set is randomly chosen. Cardinality of the chosen set is the value of random variable $Y$. Find $\mathrm{E}(X)$ i $\mathrm{E}(Y)$.

