Probability & Statistics

Problem set Nº3. Week starting March 16th

- 1. A and B are events such that $P(A \cap B) = 1/4$, $P(A^C) = 1/3$, P(B) = 1/2. Find $P(A \cup B)$.
- 2. Is it true that 13. day of the month is connected with Friday? (January 1, 1601 December 31, 2000)

EXPLANATION: Year n is a leap year if $n \equiv_4 0$, with the exception of years divisible by 100 ($n \not\equiv_{100} 0$); unless $n \equiv_{400} 0$ (i.e. year 2000). How many times in 400-year cycle 13. day of the month was Monday, Tuesday, ..., Sunday?

Random variables X, Y are independent, iff, in discrete case, condition $P(X = x_i, Y = y_k) = P(X = x_i) \cdot P(Y = y_k)$ holds.

- 3. R.v. X has binomial distribution $B(n_1, p)$ and r.v. Y $B(n_2, p)$ distribution. X, Y are independent. Prove that Z = X + Y has $B(n_1 + n_2, p)$ distribution.
- 4. Independent r.vs. X, Y have Poisson distribution with parameters λ_1 i λ_2 . Prove that r.v. Z = X + Y has Poisson distribution with parameter $\lambda_1 + \lambda_2$.

Density of r.v. (X, Y) has form f(x) = 3xy on area bounded by y = 0, y = x, y = 2 - x.

- 5. Find marginal densities $f_1(x), f_2(y)$.
- 6. Calculate expected value of Y. Check if r.v. X, Y are independent.
- 7. Probability of success in independent trials equals p. We perform trials until 3 successes occur. R.v. X is equal to number of performed trials. Find distribution of X, i.e. find density function (probabilities) and expected value X.
- 8. Readable and thoroughly without using the notes write upper and lower Greek letters: alpha α , beta β , zeta ζ , eta η , lambda λ , chi χ , xi ξ , phi ϕ , rho ρ .
- 9. (a) Let X ~ U[-2,2]. Find distribution of Y = |X|.
 (b) Given X ~ U[-1,1] find distributions of Y = X³, Z = X².
- 10. Let X be r.v. with geometric distribution $(X \sim \text{Geom}(p))$. Check that $V(X) = \frac{1-p}{p^2}$.
- 11. Cardinality of sets A_1, \ldots, A_4 is equal respectively 40, 32, 20, 50. An element (from set of 142 elements) is randomly chosen. Cardinality of the set from which chosen element was taken is the value of random variable X. Next a set is randomly chosen. Cardinality of the chosen set is the value of random variable Y. Find E(X) i E(Y).

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