Constructing recurrences for the coefficients of expansions in series of q-classical orthogonal polynomials

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Abstract

An algorithmic approach is given to construct recurrence relations for the coefficients of the series expansion of a solution of a linear q-difference equation in q-classical orthogonal polynomials (in the Hahn's sense).

Two examples are given. First, we consider the connection

$$\frac{\partial}{\partial a} P_n(x; a, b, c; q) = \sum_{k=0}^{n-1} c_k P_k(x; a, b, c; q),$$

where $P_k(x; a, b, c; q)$ are monic big q-Jacobi polynomials. Second, we discuss expansion of the basic hypergeometric function

$$f(x) = {}_3\phi_2 \left(\begin{matrix} a_1, a_2, a_3 \\ b_1, b_2 \end{matrix} \middle| q; \omega x \right)$$

with respect to q-classical orthogonal polynomials $\{p(x; q)\}$ belonging to a subclass containing a.o. little q-Jacobi, q-Krawtchouk, little q-Laguerre/Wall, alternative q-Charlier, and Stieltjes-Wigert polynomials.