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GENERALIZED BERNSTEIN POLYNOMIALS

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Abstract

We introduce *generalized Bernstein polynomials of degree* n ($n \in \mathbb{N}$),

$$B_i^n(x;\omega\,|\,q) := \frac{1}{(\omega;\,q)_n}\begin{bmatrix} n \\ i \end{bmatrix}_q x^i\,(\omega x^{-1};\,q)_i\,(x;\,q)_{n-i} \qquad (i = 0, 1, \ldots, n),$$

where q and ω are real parameters such that $q \neq 1$, and $\omega \neq 1, q^{-1}, \ldots, q^{1-n}$. Here we use the *q-Pochhammer symbol* defined for any $c \in \mathbb{C}$ by

$$(c;\,q)_0 := 1, \qquad (c;\,q)_k := \prod_{j=0}^{k-1}(1 - c\,q^j) \qquad (k \geq 1),$$

and the *q–binomial coefficient* given by

$$\begin{bmatrix} n \\ i \end{bmatrix}_q := \frac{(q;\,q)_n}{(q;\,q)_i\,(q;\,q)_{n-i}}.$$

Notice that the classical *Bernstein polynomials*

$$B_i^n(x) = \binom{n}{i} x^i (1-x)^{n-i} \qquad (0 \leq i \leq n),$$

the *discrete (Sablonnière-)Bernstein polynomials*

$$b_i^n(N, x) = \frac{1}{(-N)_n}\binom{n}{i}(-x)_i(x-N)_{n-i} \qquad (0 \leq i \leq n \leq N;\ N \in \mathbb{N}),$$

where the *Pochhammer symbol* $(c)_k$ is defined for any $c \in \mathbb{C}$ by

$$(c)_0 := 1, \qquad (c)_k := c(c+1)\cdots(c+k-1) \qquad (k \geq 1),$$

as well as the *q-Bernstein (or Phillips-Bernstein) polynomials*

$$b_i^n(x;q) = \begin{bmatrix} n \\ i \end{bmatrix}_q x^i\ (x;\,q)_{n-i} \qquad (0 \leq i \leq n),$$

are limit or particular forms of the polynomials $B_i^n(x;\omega\,|\,q)$.

Basic properties of the new polynomials are given. Also, formulas relating generalized Bernstein polynomials, big q–Jacobi and q–Hahn (or dual q-Hahn) polynomials are presented.