Dual generalized Bernstein basis

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Abstract. The generalized Bernstein basis in the space $\Pi_n$ of polynomials of degree at most $n$, being an extension of the $q$-Bernstein basis introduced recently by G. M. Phillips, is given by the formula (see S. Lewanowicz & P. Wojny, BIT 44 (2004), 63–78)

$$B_n^i(x; \omega|q) := \frac{1}{(\omega; q)_n} \left[ \begin{array}{c} n \\ i \end{array} \right]_q x^i (\omega x^{-1}; q), (x; q)_{n-i} \quad (i = 0, 1, \ldots, n).$$

We give explicitly the dual basis functions $D_n^k(x; a, b, \omega|q)$ for the polynomials $B_n^i(x; \omega|q)$, in terms of big $q$-Jacobi polynomials $P_k(x; a, b, \omega/q; q)$, $a$ and $b$ being parameters; the connection coefficients are evaluations of the $q$-Hahn polynomials. An inverse formula – relating big $q$-Jacobi, dual generalized Bernstein, and dual $q$-Hahn polynomials – is also given. Further, an alternative formula is given, representing the dual polynomial $D_n^j (0 \leq j \leq n)$ as a linear combination of min$(j, n-j) + 1$ big $q$-Jacobi polynomials with shifted parameters and argument. Finally, we give a recurrence relation satisfied by $D_n^k$, as well as an identity which may be seen as an analogue of the extended Marsden’s identity.