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Results on the associated classical orthogonal polynomials. (English summary)
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The classical orthogonal polynomials (Jacobi, Laguerre, Hermite, and Bessel polynomials) all satisfy a second-order differential equation and a three-term recurrence relation $xP_k(x) = \xi_0(k)P_{k-1}(x)+\xi_1(k)P_k(x)+\xi_2(k)P_{k+1}(x)$, with recurrence coefficients $\xi_i(k)$ which are simple rational functions of the degree k. Changing the recurrence coefficients $\xi_i(k)$ to $\xi(k+c)$ gives the associated orthogonal polynomials $P_k(x;c)$ of order c. Usually one only deals with integer order c, but for classical orthogonal polynomials any real $c>0$ is possible. In 1991, it was shown by Belmehdi and Ronveaux that the associated polynomials of order c satisfy a fourth-order differential equation $\mathbf{M}_n^{(c)}P_n(x;c)=0$. The present author shows that this differential operator is of the form $\mathbf{M}_n^{(c)}=\mathbf{M}_n^{(1)}+\theta_{n,c}\mathbf{Q}_n$, where $\theta_{n,c}$ is a real number and $\mathbf{Q}_n$ is a second-order differential operator which is independent of c. This allows the author to find a second-order recurrence relation for the connection coefficients $a_{n,k}^{(c)}$ in the relation $P_n(x;c)=\sum_{k=0}^n a_{n,k}^{(c)}P_k(x)$. The four cases (Jacobi, Laguerre, Hermite, and Bessel) are then worked out in detail.

{For the entire collection see MR1379115 (96j:00036)}

Reviewed by *Walter Van Assche*