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Structure relations for the bivariate big q-Jacobi polynomials

Stanisław Lewanowicz, Paweł Woźny, Rafał Nowak

Institute of Computer Science, University of Wrocław, ul. F. Joliot-Curie 15, 50-383 Wrocław, Poland

Abstract

The bivariate big q-Jacobi polynomials are defined by [3]

$$P_{n,k}(x,y;a,b,c,d;q) := P_{n-k}(y;a,bcq^{2k+1},dq^k;q)\, y^k (dq/y;\, q)_k\, P_k\,(x/y;c,b,d/y;q) \qquad (n \geq 0;\ k=0,1,\ldots,n),$$

where $q \in (0,\,1)$, $0 < aq$, bq, $cq < 1$, $d < 0$, and $P_m(t;\alpha,\beta,\gamma;q)$ are univariate big q-Jacobi polynomials,

$$P_m(t;\alpha,\beta,\gamma;q) := {}_3\phi_2\left(\begin{array}{c} q^{-m}, \alpha\beta q^{m+1}, t \\ \alpha q, \gamma q \end{array} \middle|\, q;q\right) \quad (m \geq 0)$$

(see, e.g., [1, Section 7.3]). We give structure relations in the form

$$\sigma_1^{\pm} D_{q^{\pm1},x}\mathbb{P}_n = F_{n,1}^{\pm}\mathbb{P}_{n+1} + G_{n,1}^{\pm}\mathbb{P}_n + H_{n,1}^{\pm}\mathbb{P}_{n-1},$$
$$\sigma_2^{\pm} D_{q^{\pm1},y}\mathbb{P}_n = F_{n,2}^{\pm}\mathbb{P}_{n+1} + G_{n,2}^{\pm}\mathbb{P}_n + H_{n,2}^{\pm}\mathbb{P}_{n-1},$$

where

$$\mathbb{P}_n := [P_{n,0},\, P_{n,1},\, \ldots, P_{n,n}]^T, \qquad P_{n,k} = P_{n,k}(x,y;a,b,c,d;q),$$
$$D_{q^{\pm1},z}\mathbb{P}_n := [D_{q^{\pm1},z}P_{n,0},\, D_{q^{\pm1},z}P_{n,1},\, \ldots, D_{q^{\pm1},z}P_{n,n}]^T \qquad (z = x,\, y),$$

$D_{q^{\pm1},x}$ and $D_{q^{\pm1},y}$ are partial q-derivative operators, $\sigma_i^{\pm}$ are certain polynomials of total degree 2, and $F_{n,i}^{\pm}$, $G_{n,i}^{\pm}$ and $H_{n,i}^{\pm}$ $(i=1,2)$ are tridiagonal matrices of appropriate dimensions. We discuss in full detail the case of the polynomials $P_{n,k}(x,y;a,b,c,0;q)$, which are closely related to Dunkl's bivariate (little) q-Jacobi polynomials [2].

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