Structure relations for the bivariate big $q$-Jacobi polynomials

Stanisław Lewanowicz, Paweł Woźni, Rafał Nowak

Institute of Computer Science, University of Wrocław, ul. F. Joliot-Curie 15, 50-383 Wrocław, Poland

Abstract

The bivariate big $q$-Jacobi polynomials are defined by [3]

$$P_{n,k}(x, y; a, b, c, d; q) := P_{n-k}(y; a, bcq^{2k+1}, dq^k; q)^k (dq/y; q)^{k} P_k(x/y; c, b, d/y; q)$$

$n \geq 0; k = 0, 1, \ldots, n)$,

where $q \in (0, 1), 0 < aq, bq, cq < 1, d < 0$, and $P_m(t; \alpha, \beta, \gamma; q)$ are univariate big $q$-Jacobi polynomials,

$$P_m(t; \alpha, \beta, \gamma; q) := \binom{q^{-m}, \alpha \beta q^{m+1}, t}{\alpha \gamma q} (m \geq 0)$$

(see, e.g., [1, Section 7.3]). We give structure relations in the form

$$\sigma_{\pm}^1 D_{q^{\pm 1}, x} P_n = F_{\pm}^x P_{n+1} + G_{\pm}^x P_n + H_{\pm}^x P_{n-1},$$

$$\sigma_{\pm}^2 D_{q^{\pm 1}, y} P_n = F_{\pm}^y P_{n+1} + G_{\pm}^y P_n + H_{\pm}^y P_{n-1},$$

where

$$P_n := [P_{n,0}, P_{n,1}, \ldots, P_{n,n}]^T, \quad P_{n,k} = P_{n,k}(x, y; a, b, c, d; q),$$

$$D_{q^{\pm 1}, x} P_n := [D_{q^{\pm 1}, x} P_{n,0}, D_{q^{\pm 1}, x} P_{n,1}, \ldots, D_{q^{\pm 1}, x} P_{n,n}]^T (z = x, y),$$

$D_{q^{\pm 1}, x}$ and $D_{q^{\pm 1}, y}$ are partial $q$-derivative operators, $\sigma_{\pm}^1$ are certain polynomials of total degree 2, and $F_{\pm}^x, G_{\pm}^x, H_{\pm}^x$ are tridiagonal matrices of appropriate dimensions. We discuss in full detail the case of the polynomials $P_{n,k}(x, y; a, b, c, 0; q)$, which are closely related to Dunkl’s bivariate (little) $q$-Jacobi polynomials [2].

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