Multi-degree reduction of tensor product Bézier surfaces with general boundary constraints

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Abstract

We propose an efficient approach to the problem of multi-degree reduction of rectangular Bézier patches, with prescribed boundary control points. We observe that the solution can be given in terms of constrained bivariate dual Bernstein polynomials. The complexity of the method is $O(m n_1 n_2)$ with $m := \min(m_1, m_2)$, where $(n_1, n_2)$ and $(m_1, m_2)$ is the degree of the input and output Bézier surface, respectively. If the approximation—with appropriate boundary constraints—is performed for each patch of several smoothly joined rectangular Bézier surfaces, the result is a composite surface of global $C^r$ continuity with a prescribed $r \geq 0$. In the detailed discussion, we restrict ourselves to $r \in \{0, 1\}$, which is the most important case in practical application. Some illustrative examples are given.

Keywords: Rectangular Bézier surface, Multi-degree reduction, Constrained dual Bernstein basis, Jacobi polynomials, Hahn polynomials.