Dual generalized Bernstein basis

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Abstract

The generalized Bernstein basis in the space \( \Pi_n \) of polynomials of degree at most \( n \), being an extension of the \( q \)-Bernstein basis introduced recently by G.M. Phillips, is given by the formula (see S. Lewanowicz & P. Woźny, BIT 44 (2004), 63–78)

\[
B_n^i(x; \omega|q) := \frac{1}{(\omega; q)_n} \left[ \begin{array}{c} n \\ i \end{array} \right] q^i (\omega x^{-1}; q)_i (x; q)_{n-i} \quad (i = 0, 1, \ldots, n).
\]

We give explicitly the dual basis functions \( D_n^k(x; a, b, \omega|q) \) for the polynomials \( B_n^i(x; \omega|q) \), in terms of big \( q \)-Jacobi polynomials \( P_k(x; a, b, \omega/q; q) \), \( a \) and \( b \) being parameters; the connection coefficients are evaluations of the \( q \)-Hahn polynomials. An inverse formula – relating big \( q \)-Jacobi, dual generalized Bernstein, and dual \( q \)-Hahn polynomials – is also given. Further, an alternative formula is given, representing the dual polynomial \( D_n^j \) \((0 \leq j \leq n)\) as a linear combination of \( \min(j, n-j) + 1 \) big \( q \)-Jacobi polynomials with shifted parameters and argument. Finally, we give a recurrence relation satisfied by \( D_n^k \), as well as an identity which may be seen as an analogue of the extended Marsden’s identity.

Key words: Generalized Bernstein basis; \( q \)-Bernstein basis; Bernstein basis; Discrete Bernstein basis; Dual basis; Big \( q \)-Jacobi polynomials; Little \( q \)-Jacobi polynomials; Shifted Jacobi polynomials.

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