Querying Best Paths in Graph Databases

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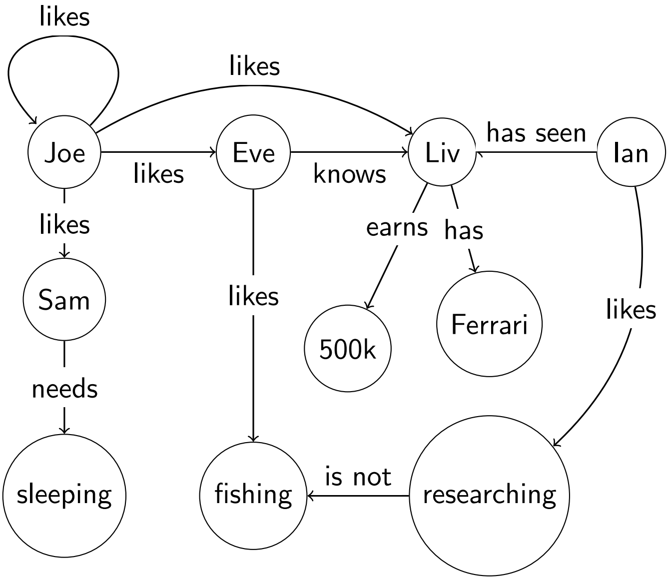
December 14, 2017

Graph Databases



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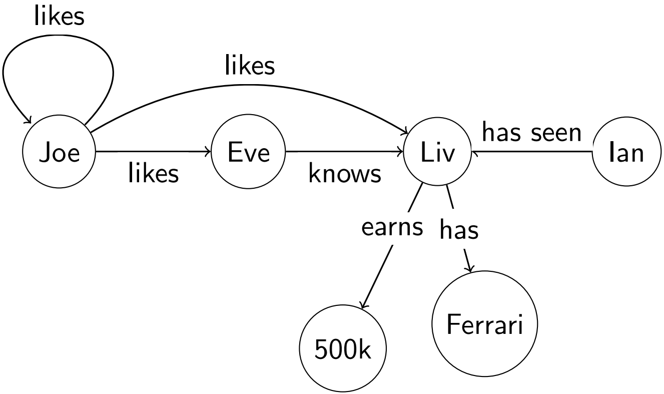

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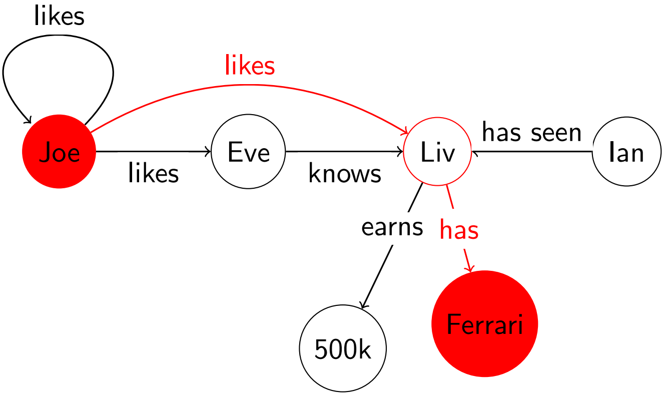

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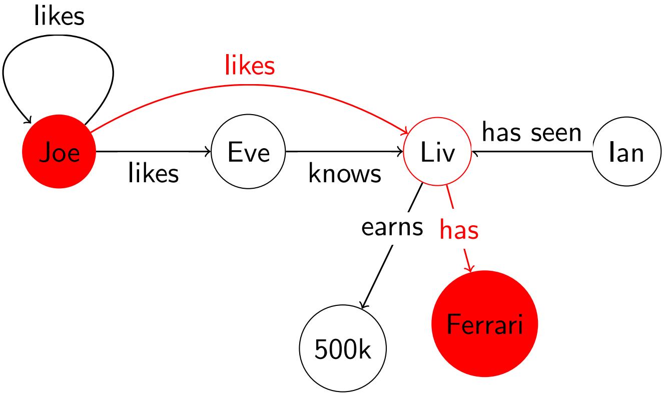

Evaluation

- PSPACE-complete (combined complexity)
- NL-complete (data complexity)

Extensions of RPQs

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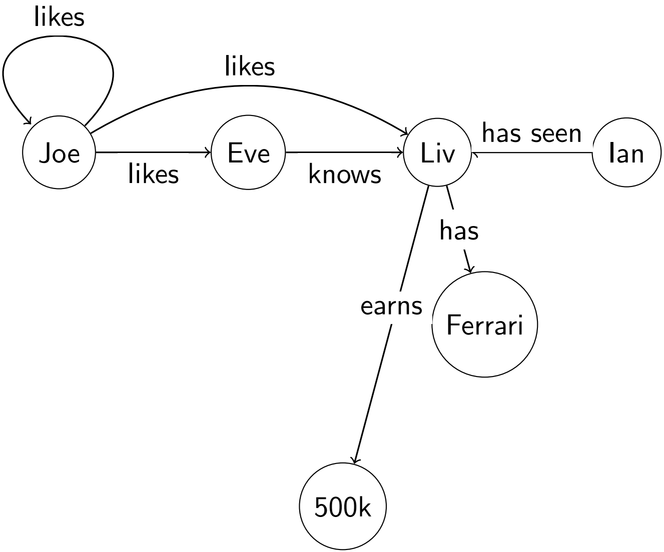
- comparing paths: ECRPQs (regular relations), +linear constraints (Barcelo, Libkin, Lin, Wood),
- data values (properties)?
  - register automata and regular queries with memory (Vrgoč and Libkin)
  - LARE (arithmetical regular expressions) (Graboń, Michaliszyn, Otop, Wieczorek)

Our approach: (O)PRA

- the comparison of paths and the use of data values,
- aggregation of data values along paths,
- computation of extremal values among aggregated data.
- modular structure (views/ontologies),
- good expressive power: subsumes earlier approaches,
- good complexity: PSPACE-complete (combined) and NL-complete (data),

Graph Databases: labellings



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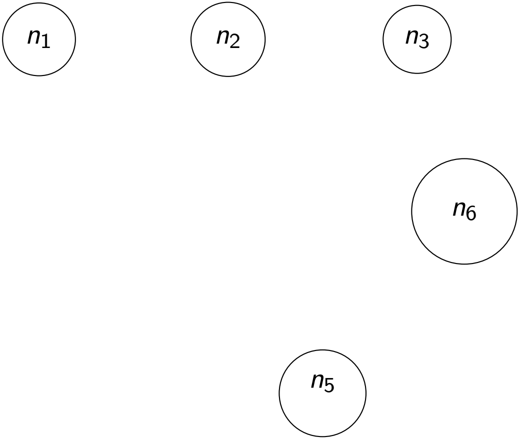

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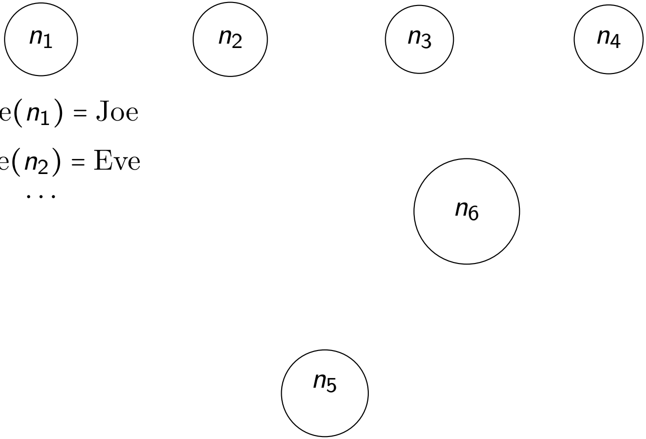

$\text{name}(n_1) = \text{Joe}$

$\text{name}(n_2) = \text{Eve}$

$\dots$

Graph Databases: labellings

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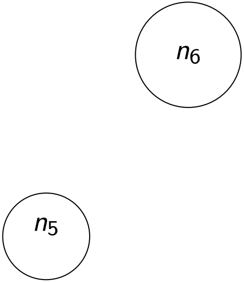
$$\dots$$

$$\text{edge}(n_1, n_2) = \mathit{true}$$

$$\text{edge}(n_2, n_3) = \mathit{true}$$

$$\text{edge}(n_1, n_5) = \mathit{false}$$

$$\dots$$



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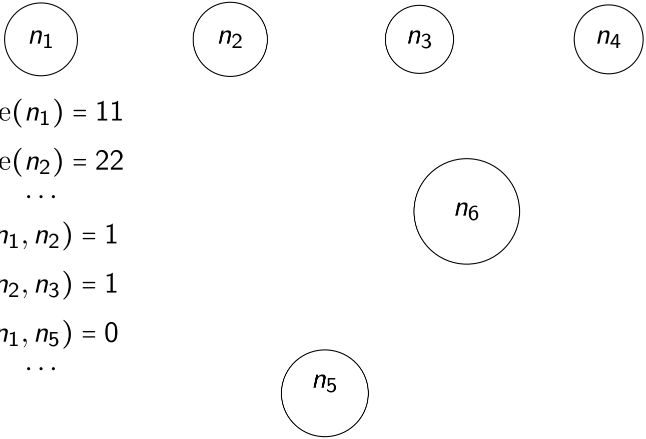

$$\mathrm{name}(n_1) = 11$$
$$\mathrm{name}(n_2) = 22$$
$$\ldots$$
$$\mathrm{edge}(n_1, n_2) = 1$$
$$\mathrm{edge}(n_2, n_3) = 1$$
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$$\ldots$$

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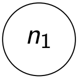

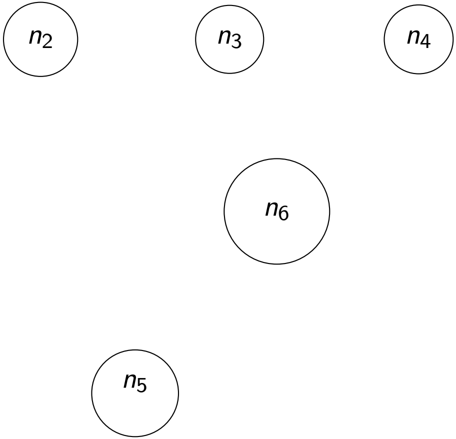

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$$\mathrm{attr}(n_3) = -108$$

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  $\mathbf{C}_1$ is the current node,
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  $\langle \mathrm{type}(\mathbf{C}_1) = 7 \rangle$
- In general: $\langle X \leq X' \rangle, \langle X < X' \rangle, \langle X = X' \rangle$
  where X, X' are integer constants or labellings applied to vars (including $\mathbf{C}_i$).

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Regular expressions over a path:

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$$\pi \qquad 1 \longrightarrow 4 \longrightarrow 8 \longrightarrow 8 \longrightarrow 9$$

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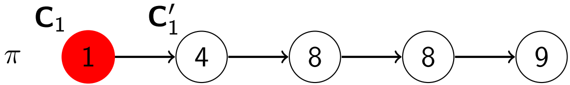

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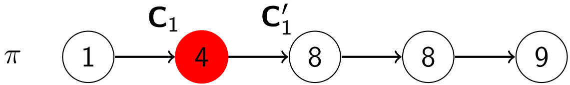

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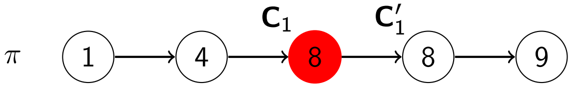

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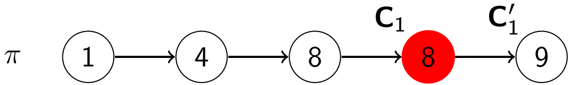

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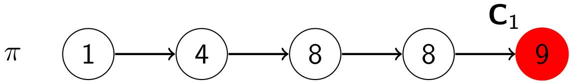

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Regular expressions over a tuple of paths (like in ECRPQs)

- Here, $\mathbf{C}_i$ is the current node on the i-th path
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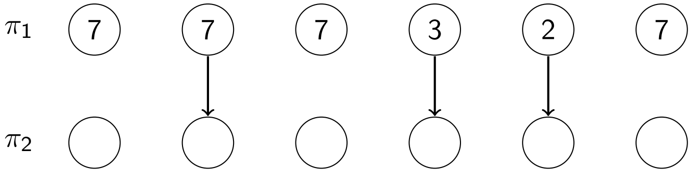

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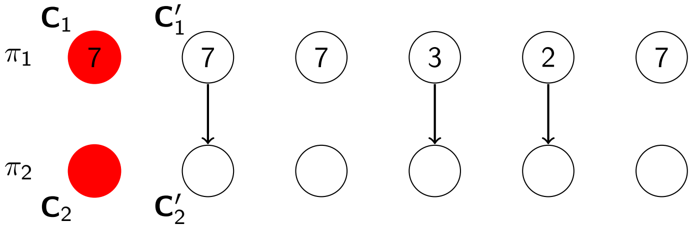

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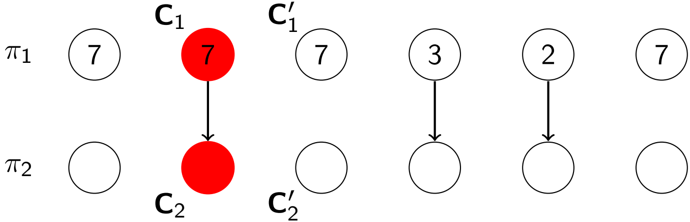

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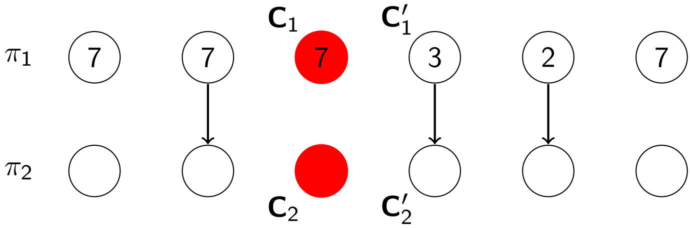

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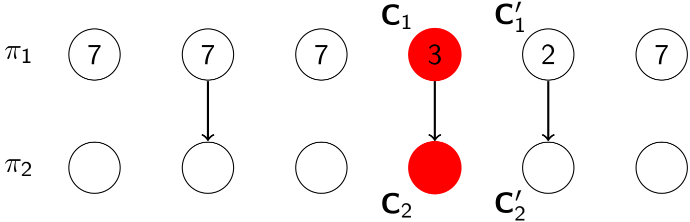

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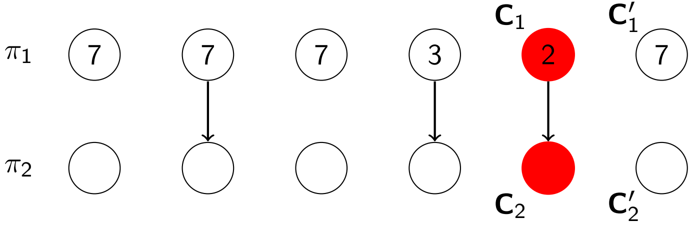

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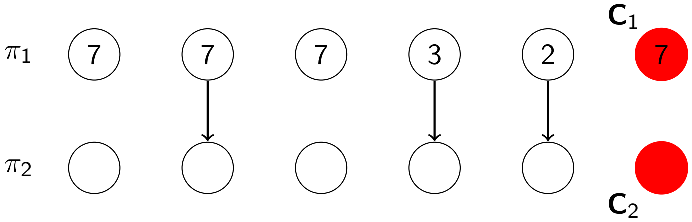

Building blocks: Arithmetical constraints

Allow to compare boolean combinations of sums along paths:

- $c_1 \Lambda_1 + \ldots + c_j \Lambda_j \leq c_0$, where c_i are constants and Λ_i is $\mathrm{some\_labelling}[\pi_{i_1}, \ldots, \pi_{i_k}]$.

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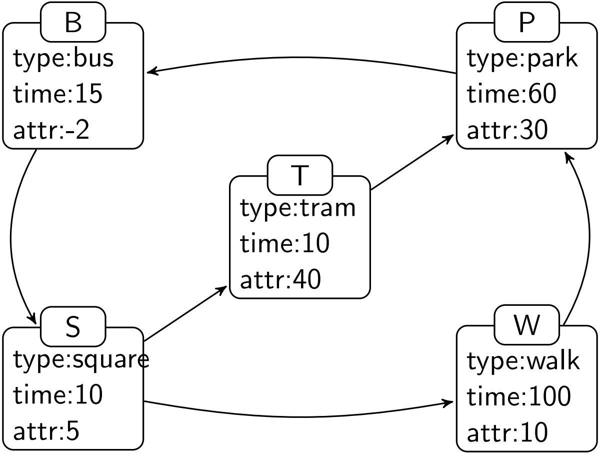
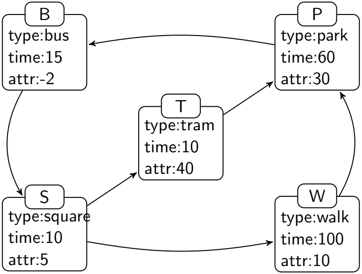
- $\mathrm{edge}[\pi_1, \pi_2] \leq 5$ (number of edges between the corresponding places in π_1 and π_2 is ≤ 5).

Example map-representing graph

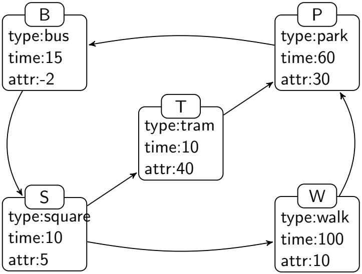


PRA Examples



$$\mathrm{route}(\pi) := \langle \mathrm{edge}(\mathbf{C}_1, \mathbf{C}_1') = 1 \rangle^* \langle \top \rangle (\pi)$$

PRA Examples



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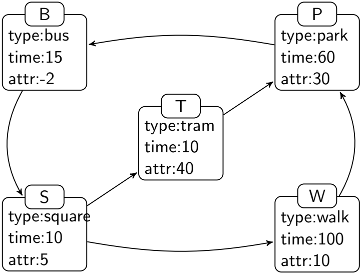
$$\texttt{MATCH NODES } (s,t) \texttt{ SUCH THAT } s \to^{\pi} t \texttt{ WHERE } \mathrm{route}(\pi)$$
$$\texttt{HAVING } \mathrm{time}[\pi] \leq 360 \wedge \mathrm{attr}[\pi] > 100$$

PRA Examples



$$
\begin{aligned}
&\texttt{MATCH NODES } (s,t) \texttt{ SUCH THAT } s \rightarrow^{\pi} t\\
&\texttt{WHERE } \mathrm{route}(\pi) \wedge \langle \mathrm{type}(\mathbf{C}_1) = c_{\mathrm{tram}} \rangle^*(\rho) \wedge\ (\langle \mathrm{type}(\mathbf{C}_1) = c_{\mathrm{bus}} \rangle +\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \langle \mathrm{type}(\mathbf{C}_1) = c_{\mathrm{walk}} \rangle +\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \langle \mathrm{type}(\mathbf{C}_1) = c_{\mathrm{tram}} \rangle +\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \langle \mathrm{edge}(\mathbf{C}_1, \mathbf{C}_2) = 1 \rangle)^*(\pi, \rho)
\end{aligned}
$$

OPRA: defining new labellings

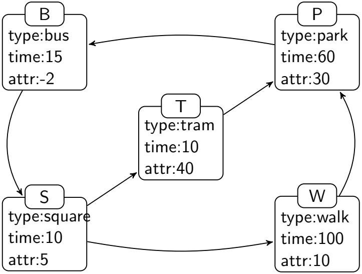
Values for new labellings are specified with *terms*.

$$t(\vec{x}) ::= c \mid \lambda(\vec{y}) \mid [Q(\vec{y})] \mid \min_{\lambda,\pi} Q(\vec{y},\pi) \mid \max_{\lambda,\pi} Q(\vec{y},\pi)$$
$$\mid y = y \mid f(t(\vec{y}),\ldots,t(\vec{y})) \mid f'(\{t(x) \colon t(x,\vec{y})\})$$

where $f, f' \in \{\textsc{Max}, \textsc{Min}, \textsc{Count}, \textsc{Sum}, +, -, \cdot, \leq$ (assuming 0 for false and 1 for true)

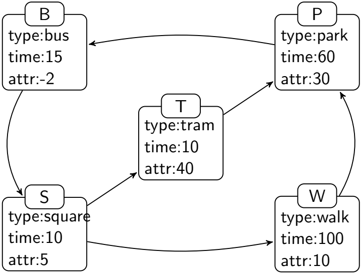
Then, we may write $\texttt{LET}\ \lambda_1{:=}t_1,\ldots,\lambda_n{:=}t_n\ \texttt{IN}\ Q$.

OPRA Example: new labelling



$\texttt{LET}\ \mathrm{walk\_time}(x) :=$
$(\mathrm{type}(x) = c_{\mathrm{walk}}) \cdot \mathrm{time}(x)\ \texttt{IN}$
$\texttt{MATCH NODES}\ (s, t)\ \texttt{SUCH THAT}\ s \to^{\pi} t$
$\texttt{WHERE}\ \mathrm{route}(\pi)\ \texttt{HAVING}\ \mathrm{walk\_time}[\pi] \leq 10$

OPRA Example: nested query



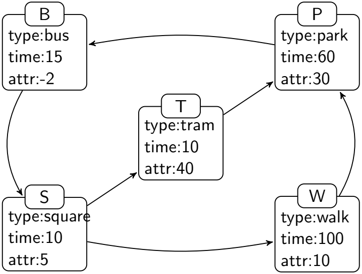
$\texttt{LET}\ \mathrm{crowded}(x) :=$

$\texttt{[MATCH NODES}\ (x)\ \texttt{SUCH THAT}\ x \to^{\pi} y\ \texttt{WHERE}$

$\mathrm{route}(\pi) \wedge \langle \top \rangle^{*} \langle \mathrm{attr}(\mathbf{C}_1) > 100 \rangle (\pi)\ \texttt{HAVING}\ \mathrm{time}[\pi] \leq 10\ \texttt{] IN}$

$\texttt{MATCH PATHS}\ (\pi)\ \texttt{WHERE}\ \mathrm{route}(\pi) \wedge \langle \mathrm{crowded}(\mathbf{C}_1) = 0 \rangle^{*} (\pi)$

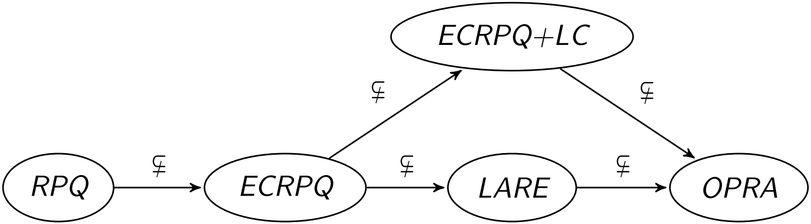
OPRA Example: most attractive but in minimum time



MATCH NODES (s, t) SUCH THAT $s \rightarrow^{\pi} t$ WHERE $\mathrm{route}(\pi)$
HAVING
$(\mathrm{attr}[\pi] = \max_{\mathrm{attr},\rho} Q_{\mathrm{route}}(s, t, \rho)) \wedge$
$(\mathrm{time}[\pi] = \min_{\mathrm{time},\rho} Q_{\mathrm{route}}(s, t, \rho))$

Our results

Theorem (Expressivity)



Theorem (Complexity)

Query answering for OPRA is PSPACE-complete (combined complexity) and NL-complete (data complexity).