1. Transform the following sequents into clauses: After that, show that the resulting clause sets are unsatisfiable.

(a) \( A, \neg A \vdash B \).

(b) \( \vdash A \lor \neg A \). (This is the famous tertium non datur principle, a third is not given.)

(c) \( \neg \neg A \vdash A \). (Double negation law.)

(d) \( \vdash (A \rightarrow B) \lor (B \rightarrow A) \).

(e) \( A \rightarrow B, \neg C \rightarrow \neg B \vdash A \rightarrow C \).

(f) \( \vdash (\neg A \lor B) \leftrightarrow (A \rightarrow B) \).

(g) \( \vdash (\neg A \land \neg B) \leftrightarrow (A \lor B) \). (De Morgan law.)

(h) \( \vdash (\neg A \lor \neg B) \leftrightarrow (A \land B) \). (De Morgan law.)

(i) \( \neg A \lor \neg B, \neg \neg A, \neg A \land B \vdash \bot \).

2. A few questions about resolution:

Which of the following clause sets are satisfiable? If the clause set is satisfiable, then find a model. If the clause is not satisfiable, then find a resolution refutation.

(a) \( \{A, B\}, \{\neg A, B\}, \{\neg B\} \).

(b) \( \{A, B\}, \{\neg A, B\}, \{A, \neg B\}, \{\neg A, \neg B\} \).

(c) \( \{\neg A, B\}, \{A, C, B\}, \{B, C\}, \{B, \neg C\} \).

(d) \( \{A, B, E\}, \{\neg A, C\}, \{\neg B, D, E\}, \{\neg A, \neg C\}, \{\neg B, \neg D\}, \{\neg E\} \).

3. Resolution remains complete, when it is controlled by priorities. Assume that we have the following priorities on atoms. (1) \( A \), (2) \( B \), (3) \( C \), (4) \( D \), and (5) \( E \).

Resolution must always use the literal with the highest priority. That means that constructing \( (C_1\setminus\{B\}) \cup (C_2\setminus\{\neg B\}) \) is only possible when \( C_1, C_2 \) do not contain \( C, D \) or \( E \).

Using resolution with priorities, find a resolution refutation of the clause set of task 2.d.