1 Exercises for the Lecture Automated Theorem Proving

(To be completed by May 31st, 18.00)

1. Let the $A$-order $\succ$ be defined by:

$$P_1 \succ P_2 \succ A \succ Q_1 \succ Q_2.$$ 

Let the selection function $\Sigma$ be defined by:

$$\Sigma(c) = \neg Q_1 \text{ if } (\neg Q_1) \in c, \; \emptyset \text{ otherwise.}$$

Consider the clause set $S =$

$$
\begin{align*}
&[P_1, \neg Q_1] & 1 \\
&[\neg P_1, Q_1] & 2 \\
&[\neg P_1, \neg Q_1] & 3 \\
&[P_2, Q_2, \neg A] & 4 \\
&[P_2, \neg Q_2] & 5 \\
&[\neg P_2, Q_2] & 6 \\
&[\neg P_2, \neg Q_2] & 7 \\
&[\neg A, Q_2] & 8 \\
&[\neg Q_2, \neg Q_2] & 9 \\
&[Q_2, \neg Q_2] & 10 \\
&[\neg Q_2] & 11
\end{align*}
$$

Convince yourself of the fact that $S$ is a saturated set, and that it is a saturation of the clauses 1, \ldots, 7.

(Actually, $S$ would already have been a saturated set without 9 and 10. Why?)

2. Rank the clauses of $S$.

3. Perform the model construction for $S$.

4. Show that for the ranking function the following holds: If $c_1$ is ranked before $c_2$, and $c_2$ is ranked before $c_3$, then $c_1$ is ranked before $c_3$. (This was implicitly used in the completeness proof. Without it, the ranking function would be useless as a ranking function)

5. Show that: If $c_1$ is ranked before $c_2$, then $c_1 \cup d$ is ranked before $c_2 \cup d$. ($\cup$ denotes multiset union)

6. (The following fact was used in the completeness proof, but not really proven)
Let \( R_1 \cup R_2 \) be the resolvent of \( \neg A \cup R_1 \) with \( A \cup R_2 \). Assume that \( A \succ R_2 \). Show that \( R_1 \cup R_2 \) has a lower rank than \( \neg A \cup R_1 \). (Decompose the problem, s.t. the claims in the two previous questions can be used)

7. Given the \( A \)-order of the first question, do \( \neg P_1, [P_1] \) make \( [P_2] \) redundant?
   Do \( \neg P_2, [P_2] \) make \( [P_1] \) redundant?
   Does \( [\ ] \) make \( [P_1, \neg Q_1] \) redundant?
   Do \( [P_2, Q_2, \neg A], [P_2, \neg Q_2], [\neg P_2, Q_2], [\neg P_2, \neg Q_2] \) make \( \neg A \) redundant?
   And \( \neg A, \neg A \)?