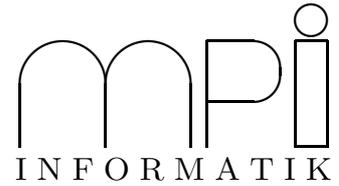




## Interactive Proof Tools Assignment 7

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<http://www.mpi-sb.mpg.de/~nivelle/teaching/intprooftools2003/main.html>

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The exercises 7.1 and 7.2 belong to our project of formalizing and verifying a propositional resolution calculus in PVS.

**Exercise 7.1** From formulas to sets of clauses:

1. Formalize propositional formulas (constructed from propositional variables, negation, conjunction, disjunction, implication and equivalence), and define the predicates `true?`, `satisfiable?` and `valid?` for formulas.

**Hint:** Define formulas as an abstract datatype.

2. Define an equivalence transformation from formulas to finite sets of clauses, i. e., a transformation that converts a propositional formula  $\varphi$  into an equivalent finite set of clauses  $\Gamma$ . Prove, for all interpretations  $I$ , that  $I \models \varphi$  iff  $I \models \Gamma$ .

**Hints:**

- Implement the 3 transformations from slide 4 (in the right order).
- For each transformation, formalize its range (and the domain of the next transformation) as predicate subtype of the type of formulas.
- For each transformation, define the transformation as a recursive function and establish that the transformation preserves truth. Note that termination of the recursion may be rather tricky for the transformation from NNF to CNF.
- Transform a CNF formula to a finite set of clauses and prove that this transformation preserves truth.
- Compose all 4 transformations.

**Exercise 7.2** The resolution calculus Res:

1. Formalize the inferences rules **Resolution** and **Factoring**.

**Hint:** An inference rule with  $n$  premises can be viewed as an  $n + 1$ -ary relation.

2. For all (not necessarily finite) clause sets  $\Gamma$ , define the sets  $\text{Res}(\Gamma)$  and  $\text{Res}^*(\Gamma)$ .

**Hint:** Use an inductive definition for  $\text{Res}^*(\Gamma)$ .

3. Prove the soundness of Res.

**Exercise 7.3** Show (on paper) that  $\text{Res}^*(\Gamma)$  need not be finite for finite clause sets  $\Gamma$ .<sup>1</sup>

**Hint:** It suffices to consider a satisfiable  $\Gamma$  containing 2 clauses only.

**Exercise 7.4** Formally check (on paper) the types of the combinator terms I, K,  $K^*$  and S (see slide 18) by building their type derivation trees.

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<sup>1</sup>Contrary to what I told you in the lecture.