1. On the slides about Higher-Order logic, two alternative definitions of $=$ are given.

First consider the first definition:

$$= : = \lambda D: \text{Type} \lambda d_1, d_2: D$$

$$\forall P: D \to \text{Prop} \ (P \ d_1) \to (P \ d_2): \Pi D: \text{Type} \ D \to D \to \text{Prop}$$

(a) Show, using the type derivation rules for HOL, that the defining term of $=$ has the indicated type.
(b) Prove in natural deduction that $=$ is reflexive:

$$\forall D: \text{Type}\forall d: D \ (= D d d).$$

(c) Prove in natural deduction that $=$ is symmetric:

$$\forall d: D \forall d_1, d_2: D \ (= D d_1 d_2) \to (= D d_2 d_1).$$

(The definition of $=$ is the replacement rule. In usual natural deduction, symmetry for equality can be derived using the replacement rule. Transform this proof into a higher-order logic proof)

2.

$$=_2 : = \lambda D: \text{Type} \lambda d_1, d_2: D$$

$$\forall P: D \to D \to \text{Prop} \ (\forall d: D \ (P \ d)) \to (P \ d_1 d_2):$$

$$\Pi D: \text{Type} \ D \to D \to \text{Prop}$$

Write $=_1$ for the first definition of equality. Write $=_2$ for the second.

(a) Prove that

$$\forall D: \text{Type} \forall d_1, d_2: D \ (=_1 D d_1 d_2) \to (=_2 D d_2 d_1).$$

(b)

$$\forall D: \text{Type} \forall d_2, d_1: D \ (=_2 D d_2 d_1) \to (=_1 D d_1 d_2).$$