Introduction to Flight Simulation (List 9)

Due: 14 december 2010

1. Write a function that multiplies quaternions.

2. Write a function that computes the norm $|q|$ of a quaternion.

3. Verify for a couple of quaternions, that

$$\frac{qq^*}{|q|^2} = 1.$$ 

4. Implement the function $F_q(x) = q.x.q^{-1}$. Verify for a couple of different quaternions of form $x = (0, \mathbf{v})$ that the real component of $F_q(x) = 0$.

5. We are going to verify (by examples) that $F_q(x)$ represents a rotation. Try out that, for quaternions $x$ with empty real components, that

(a) $|F_q(x)| = |x|.$

(b) $F_q(x_1) \times F_q(x_2) = F(x_1 \times x_2).$ The $\times$-operator means crossproduct, because $x_1, x_2$ have their real components 0. The real component of $q$ does not have to be 0.

(c) By trying, find the axis of the rotation, i.e. vectors $x$, for which $F_q(x) = x$.

(d) The angle of rotation can be found by taking a unit $\mathbf{v}$ vector parallel to the axis, and using

$$\cos(\phi) = \mathbf{v}.F_q(\mathbf{v}).$$

Try to determine the angle $\phi$ for given $q$. 

1