Introduction to Flight Simulation (List 1)

Due: 12 October 2010

1. We are going to numerically approximate some integrals. The integral

$$\int_a^b f(x)\ dx$$

can be approximated by the following program

```
double sum = 0.0;
double h = some small number (for example 1E-4)
double x = a;
while( x + h < b )
{
   sum += f(x) * h;
   x += h;
}
sum += f(x) * ( b - x );
   // Because b-a need not be a multiple of h.
```

Use this scheme to approximate the natural logarithm:

$$\log(y) = \int_1^y \frac{1}{x}\ dx.$$

Compare the values of your program with the values computed by the standard function `log`. It is defined in `cmath`. Analyze how the difference between the results of your program, and the standard function `log` depends on the value of h. Explain the behaviour.

2. The accuracy of numeric integration can be improved as follows:

```
double sum = 0.0;
double h = some small number
double x = a;
while( x + h < b )
{
```

```
        sum += 0.5 * ( f(x) + f(x+h) ) * h;
        x += h;
    }
    sum += 0.5 * ( f(x) + f(b) ) * ( b - x );
```

Implement this integration scheme for the natural logarithm.

Do the same analysis as in Task 1. Is the improved method indeed better?

3. Also try Simpson's method:

```
    double sum = 0.0;
    double h = some small number
    double x = a;
    while( x + h < b )
    {
        sum += ( f(x) + 4.0 * f( x + 0.5 * h ) + f( x + h )) * h / 6.0;
        x += h;
    }
    sum += ( f(x) + 4.0 * f( 0.5 * ( x + b )) + f(b)) * ( b - x ) / 6.0;
```

Do the same analysis as in Task 1.

4. Numerically find the real zeroes of the following polynomials:

$$x^2 - x + 1 = 0,$$
$$x^3 - 4x^2 + 8x - 15 = 0,$$
$$x^4 - x^3 + x - 7 = 0.$$

The easiest way to do this is by using the Newton-Rhapson method. Suppose that we want to find zeroes of function f, which is differentiable. Let x_1 be some first guess. Then for each $i \geq 1$,

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}.$$

Continue computing successive x_i, until $f(x_i)$ is sufficiently close to 0.