How to Compute the Forces on an Aircraft

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1 Introduction

In this text, I describe what I know about the computation of aerodynamic forces, which is unfortunately not much. We intend to represent the airplane by a *rigid object*, even though airplanes in reality are not completely rigid. We have a formula that allows us to solve angular acceleration of the airplane from its inertia matrix, the angular speed, the total force, and the total torque on the airplance.

It is always important to realize in which coordinate system one is working. Simulation involves a lot of transformations of coordinates between different coordinate systems.

The outside coordinate system is the coordinate system in which specifies the scenery objects. (Landscape, navigation beacons, airports, etc.) Reasonable choices are ECEF-coordinates for around-the-earth navigation or LENU-coordinates for small sceneries, for example for practicing landings on a known airport.

Airplane coordinates are fixed to the airplane. The standard coordinate system has positive X pointing to the front of the plane, positive Y to the port side (left) of the plane, positive Z downward.

In addition to these two coordinate systems, it may be useful to define separate coordinate systems for each of the wings, the rudder, and the ailerons.

The inertia matrix of an airplane usually does not change abruptly over time. (Unless one simulates a plane that drops bombs or parachutists.) However, the weight of an airplane that flies a long distance may consist for one third of fuel. During the flight, this fuel gets burned, so it may be useful to recompute the mass and the inertia matrix of the plane from time to time.

If one wants to make use of the fact that the inertia matrix does not change much, one has to take into accout that this is only true when the inertia matrix is expressed in airplane coordinates.

Let q be a quaternion representing the orientation of the airplane, Let M_q be the matrix defined by

$$M_q(v) = q.(0; v).q^{-1},$$

for each vector v. It is the matrix that represents the rotation that is represented by q.

Let M_I be the inertial matrix of the airplane in airplane coordinates. Then the inertial matrix in external coordinates M'_I is equal to

$$M'_I = M_q.M_i.M_q^{-1}.$$

Note that M_q is an orthogonal matrix, so that its inverse M_q^{-1} can be computed by taking its transposition M_q^T.

When one computes forces on the airplane, one has to be very careful in which coordinate system the forces are represented. Gravity and forces related to collisions or contacts are naturally represented in external coordinates. Aerodynamic forces and forces from the engines are naturally represented in internal coordinates.

A force can be transformed from internal to external coordinates, simply by rotating it:

$$\overline{F} = q.(0;\overline{F}).q^{-1}, \text{ or } \overline{F} = M_q(\overline{F}).$$

2   Aerodynamic Forces

Aerodynamic forces are traditionally decomposed into *lift* and *drag.* The lift is the component of the force that is perpendicular to the air flow. The drag is the component of the force in the direction of the airflow (resistance.) For an airplane designer, the distinction between lift and drag is very important. For a wing, one wants to have much lift and little drag, because it will result in a more efficient airplane. For airplane simulation, the distinction is not important, because one has to add lift and drag into a single force anyway.

The directions of lift and drag are defined by the direction of the air flow. Since wings change the air flow around them, one has to be somewhat careful with this definition: It is the direction of airflow at sufficient distance from the wing. If there is no wind, one take take the flying direction of the airplane as reference.

3   Uncertanties

Realistic aerodynamic data are very hard to get. Until now, I know no better way than applying the generic formulas from text books. This is probably fine for flying at low speeds (Say up to Mach 0.5), but modern passenger typically fly at Mach 0.73. At such high speeds, the standard formulas do not apply anymore, so one should at least have a better formula. If one wants to simulate military planes that fly at even higher speeds, I have completely no idea from to get data.

An airplane does not only consist of wings. It also has a fuselage, which also has lift and resistance. The aerodynamic forces on the fuselage can be computed in a similar way as the forces on the wings of the plane, but it is again very difficult to get realistic data. For the fuselage of the plane, the lift does not

need to have an upward direction. If the plane slips sideways through the air, the lift of the fuselage will be in the direction opposite to the slipping direction.

Another big uncertainty is the behaviour of the engines. It is in principle easy to simulate the engine if one doesn't care too much about details. One knows the position of the engines, and the maximal thrust for zero speed at sea level is usually published. Unfortunately, I don't know how maximal thrust changes with speed and altitude. One has to guess, and try to find hints in texts. It is also important to know how thrust depends on fuel use, because otherwise one would get unrealistic ranges for the simulated airplane. On the instruments, the pilot usually sees fuel flow and EPR (Engine Pressure Ratio), which is the quotient of the pressures behind and before the main fan. If one estimates the size of the fan, and knows the outide pressure, one can compute the engine force from the EPR, but one still has to guess how EPR depends on fuel flow and speed and altitude. Modern jet engines are kind of hybrid. The main force is generated by the main fan, which can be viewed as a big propellor in a tube. The fan of a modern jet engine has quite long reaction times. When power is increased from idle to maximal, it may take up to 7 second before the engine generates its maximal force. This behavior may be important for practicing go-arounds, or avoidance of obstacles, so one should model this in the simulation. Fortunately, it can be easily modelled by a circuit, but again, the exact numbers are hard to get.

The final uncertainty that we have to deal with is the mass distribution. It is easy to know the weight of the airplane for most of the common types, but it is not so easy to find out how the weight is distributed through the plane. This information is needed for computing the inertial matrix. However, one can use the information that one knows. One can determine the maximum fuel load, and determine where the fuel tanks are. One can determine the maximal number of passengers and determine where they sit. Similarly, it is possible to determine where the cargo areas are. The distribution of the rest of the weight, which is the weight of the airplane itself, is hard to guess.

4 Basic Formula for Aerodynamic Forces

It seems that all aerodynamic forces are determined by a formula of the following form
$$\overline{F} = \frac{1}{2}\rho V^2 \overline{C}.$$
This is probably not true for speeds above Mach 0.5, but I do not know a better formula. In the formula, ρ is the density of the air, V is the speed (as scalar) and $\overline{C}$ is a coefficient that depends on the form of the object, and its attitude in the air flow. For wings, the formula assumes the following form:
$$\overline{L} = \frac{1}{2}\rho V^2 C_L(\alpha)S, \quad \overline{D} = \frac{1}{2}\rho V^2 C_D(\alpha)S.$$
Here ρ is the density of air, which is 1.225kgm^{-3} at sea level, V is the speed (as scalar), S is the surface area of the wing and $C_L(\alpha), C_D(\alpha)$ are numbers that

depend on the angle of attack α.

The table below gives the values for CLARK YH: The fourth entry in the table is the acting point of the force (needed for computing torque), relative to the chord of the wing. A number of 0 would mean that the forces acts completely on the front of the wing, a number of 1.0 would mean that the force acts completely at the end of the wing. One can see from the table that the acting point of the lift tends around one third of the wing.

α	C_L	C_D	CP
-4	-0.09	0.010	$-$
-2	0.05	0.009	0.74
0	0.20	0.010	0.40
2	0.36	0.015	0.32
4	0.51	0.022	0.295
6	0.66	0.033	0.285
8	0.80	0.045	0.275
10	0.94	0.062	0.27
12	1.06	0.083	0.27
14	1.21	0.103	0.27
16	1.33	0.125	0.265
18	1.43	0.146	0.265
19	1.36	0.170	0.275
20	1.26	0.211	0.29
25	0.97	0.324	0.33
30	0.81	0.430	0.37

It must be observed that the angle of attack and the speed can vary of the wing. If the plane is banking to the left, the angle of attack on the left wing is higher than the angle of the attack on the right wing. At low angles of attack, this will dampen the roll. At high angles of attack, this may increase the roll because the left wing could stall. In order to obtain an accurate simulation, one would divide the wing in small parts and compute the forces seprately for each part. We already know that for each mass point of a rigid object,

$$\overline{v}_i = \overline{v} + \overline{\omega} \times (\overline{x}_i - \overline{x}).$$

This equation is based on external coordinates. The position of mass point i is fixed in internal coordinates. Let us call the position $\overline{y}_i$, so that we can easily compute

$$\overline{x}_i - \overline{x} = M_{\overline{q}}(\overline{y}_i),$$

where q is the orientation of the plane.

We now know how to compute $V = |\overline{v}_i|$. In order to determine α, we need to know how the wing is connected to the airplane. It is usually connected with a fixed angle of attack, and the wings usually bend upwards. This is called *the dihedral angle*. It is important for lateral stability, so it has to be taken into account.

The easiest solution is to represent the dihedral angle and the fixed angle of attack by a quaternion. Let us call this quaternion q_w. Then the external speed vector $\overline{v}_i$ needs to be rotated by $(q.q_w)^{-1}$ and inverted, so the result is

$$\overline{v'}_i = -(q.q_w)^{-1}.(0; \overline{v}_i).(q.q_w).$$

The vector $\overline{v}'_i$ can be written in the form (x, y, z), from which the angle of attack can be calculated by computing

$$\alpha = \arctan \frac{z}{\sqrt{x^2 + y^2}}.$$

Since one is interested in $C_L(\alpha)$, which is given by a table, one can put $\frac{z}{\sqrt{x^2+y^2}}$ in the table instead of α, so that one doesn't have to compute the arc tangent all the time.

5 Interpolation

Something needs to be written about interpolation. The best way to do this seems to be *spline interpolation.*