Functional Programming

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Lecture 2: Algebra, Fig. 1

Type inference example derivation

$$\frac{[?]}{\texttt{fun x -> ((+) x) 1}:[?]}$$

use $\rightarrow$ introduction:

$$\frac{\dfrac{[?]}{\texttt{((+) x) 1}:[?\alpha]}}{\texttt{fun x -> ((+) x) 1}:[?] \to [?\alpha]}$$

use $\rightarrow$ elimination:

$$\dfrac{\dfrac{\dfrac{[?]}{\texttt{(+)}\ \texttt{x}\colon [?\beta] \to [?\alpha]} \quad \dfrac{[?]}{\texttt{1}\colon [?\beta]}}{\texttt{((+)}\ \texttt{x)}\ \texttt{1}\colon [?\alpha]}}{\texttt{fun x -> ((+) x) 1}\colon [?] \to [?\alpha]}$$

we know that `1: int`

$$\dfrac{\dfrac{\dfrac{[?]}{\texttt{(+) x}\colon \texttt{int} \to [?\alpha]} \quad \dfrac{}{\texttt{1}\colon \texttt{int}}\,(\text{constant})}{\texttt{((+) x) 1}\colon [?\alpha]}}{\texttt{fun x -> ((+) x) 1}\colon [?] \to [?\alpha]}$$

application again:

$$
\dfrac{\dfrac{\dfrac{\dfrac{[?]}{\texttt{(+)}\colon [?\gamma] \to \texttt{int} \to [?\alpha]} \quad \dfrac{[?]}{\texttt{x}\colon [?\gamma]}}{\texttt{(+) x}\colon \texttt{int} \to [?\alpha]} \quad \dfrac{}{\texttt{1}\colon \texttt{int}}\,(\text{constant})}{\texttt{((+) x) 1}\colon [?\alpha]}}{\texttt{fun x -> ((+) x) 1}\colon [?] \to [?\alpha]}
$$

it's our x!

$$
\dfrac{\dfrac{\dfrac{\dfrac{[?]}{\texttt{(+)}\colon [?\gamma] \to \texttt{int} \to [?\alpha]} \quad \dfrac{}{\texttt{x}\colon [?\gamma]}\,\texttt{x}}{\texttt{(+) x}\colon \texttt{int} \to [?\alpha]} \quad \dfrac{}{\texttt{1}\colon \texttt{int}}\,(\text{constant})}{\texttt{((+) x) 1}\colon [?\alpha]}}{\texttt{fun x -> ((+) x) 1}\colon [?\gamma] \to [?\alpha]}
$$

but $(+) : \texttt{int} \to \texttt{int} \to \texttt{int}$

$$
\dfrac{\dfrac{\dfrac{}{\texttt{(+)}\colon \texttt{int}\to\texttt{int}\to\texttt{int}}\,(\text{constant}) \qquad \dfrac{}{\texttt{x}\colon\texttt{int}}\,\texttt{x}}{\texttt{(+) x}\colon\texttt{int}\to\texttt{int}} \qquad \dfrac{}{\texttt{1}\colon\texttt{int}}\,(\text{constant})}{\texttt{((+) x) 1}\colon\texttt{int}}
$$

$$
\texttt{fun x -> ((+) x) 1}\colon\texttt{int}\to\texttt{int}
$$