

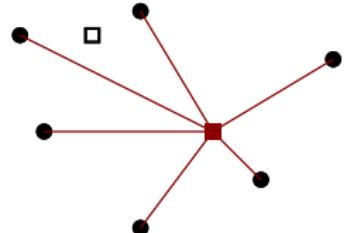
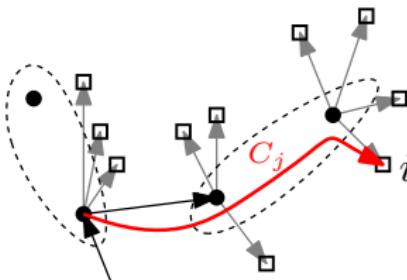
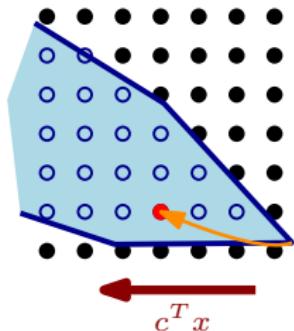
Approximation Algorithms for Multiwinner Elections and Clustering Problems



Krzysztof Sornat

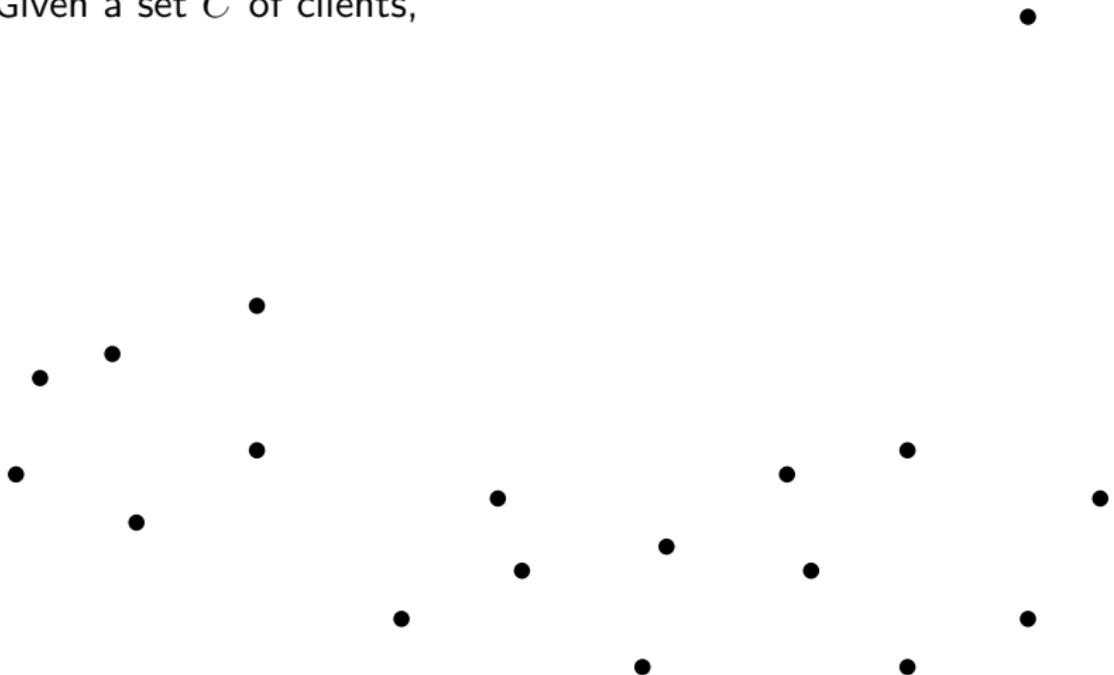
University of Wrocław, Poland

10.10.2019



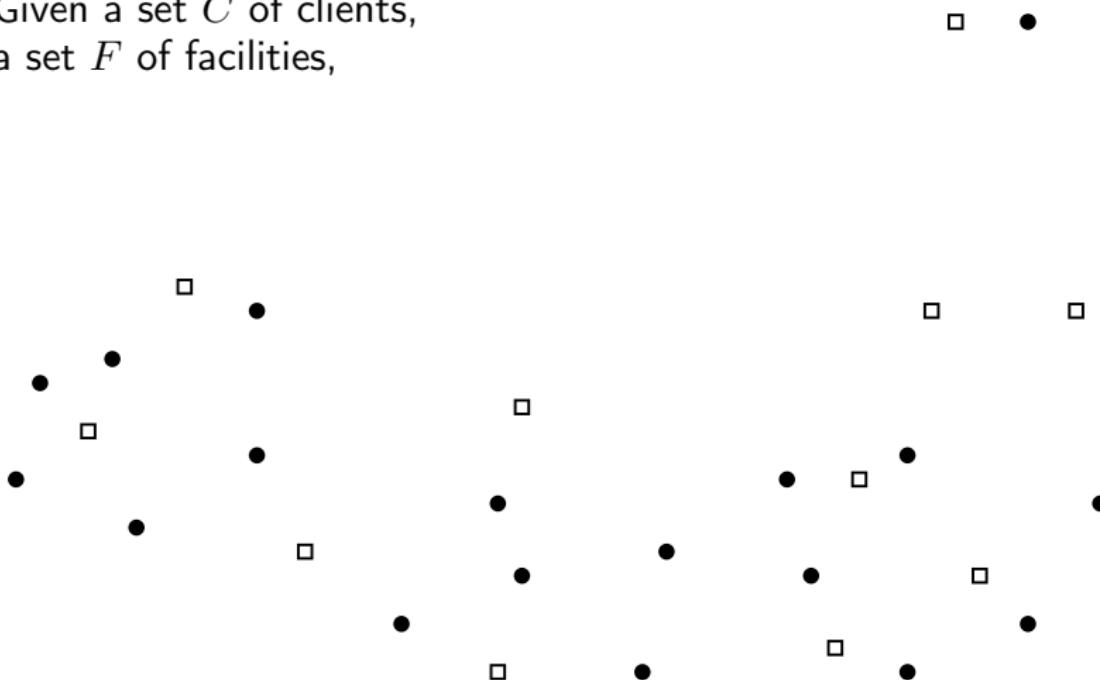
Location Problem

Given a set C of clients,



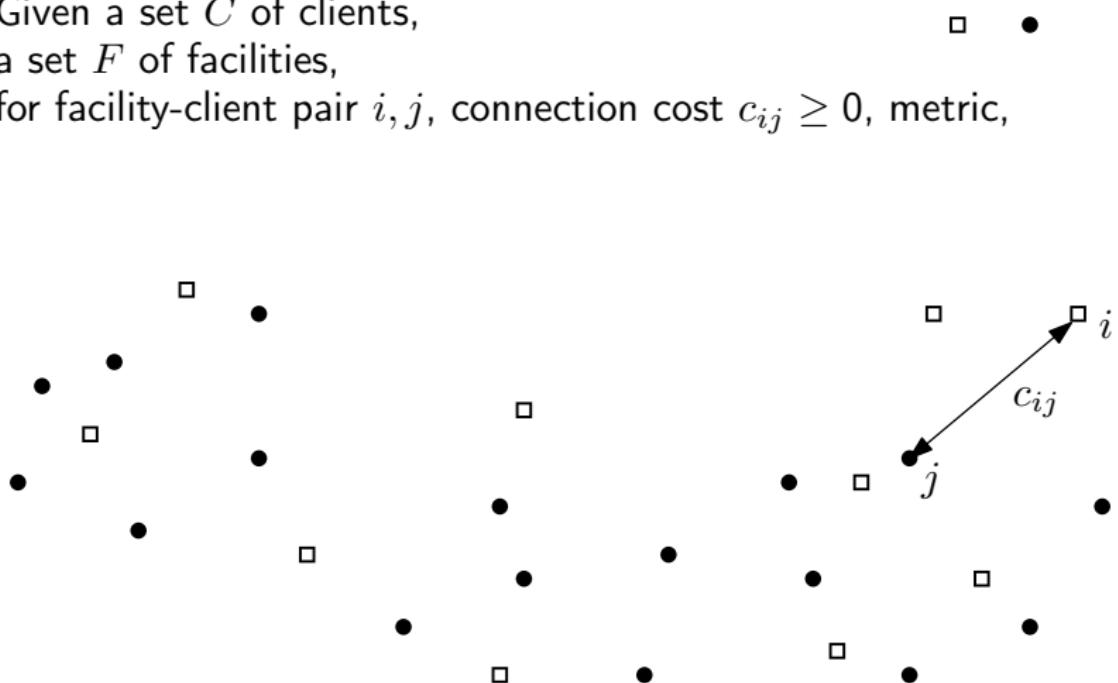
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Given a set C of clients,
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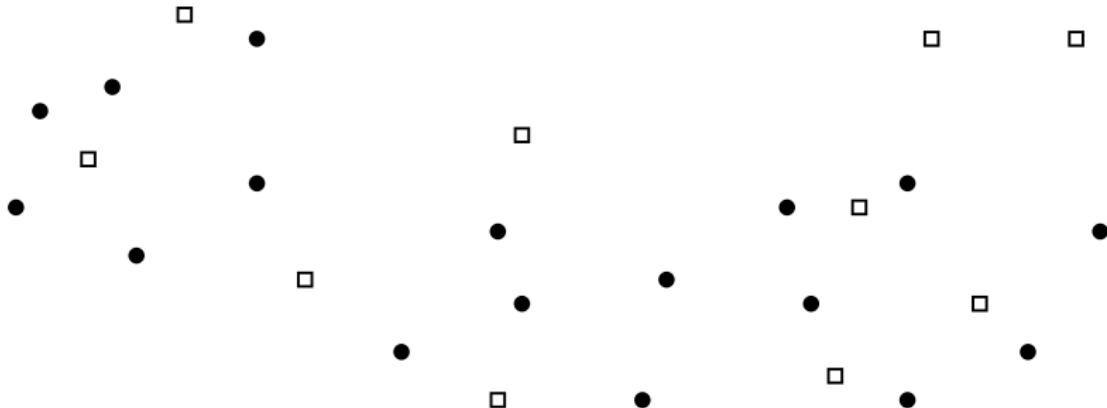
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Given a set C of clients,
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for facility-client pair i, j , connection cost $c_{ij} \geq 0$, metric,



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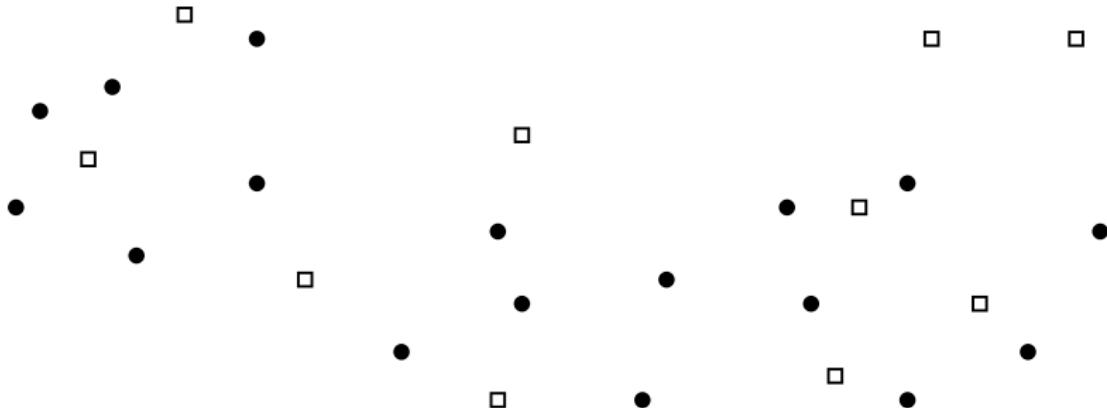
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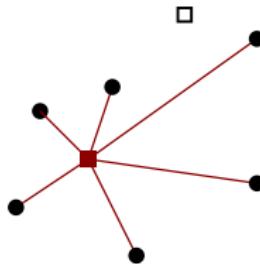
Open a set S of k facilities to serve clients “optimally”!



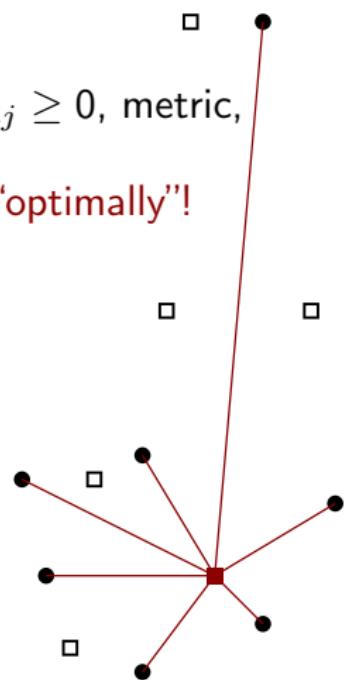
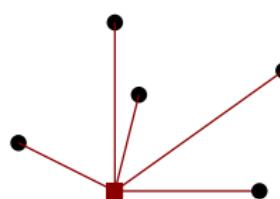
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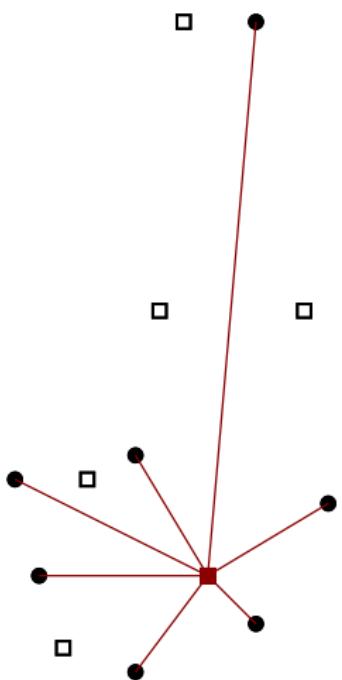
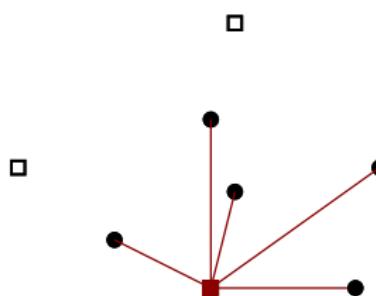
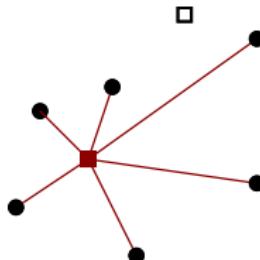


$$k = 3$$



k -MEDIAN

Think of libraries, schools, or warehouses.

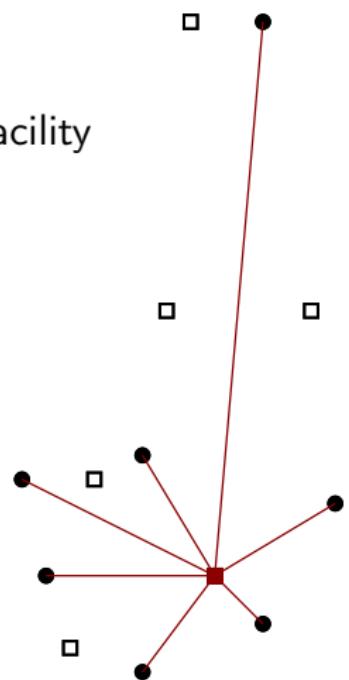
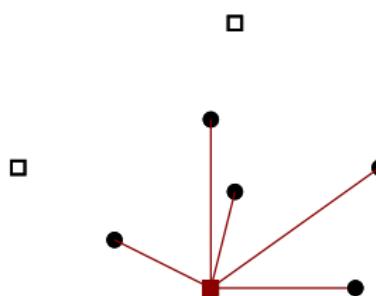
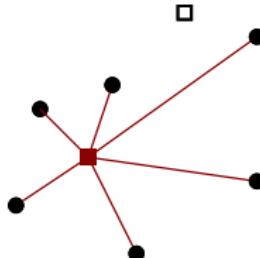


k-MEDIAN

Think of libraries, schools, or warehouses.

minimize **total** connection cost

$\sum_{j \in C} c_{i,S}$ of all clients to their closest open facility



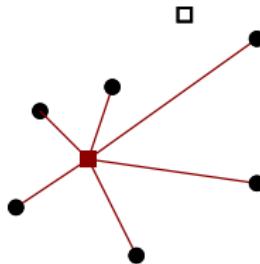
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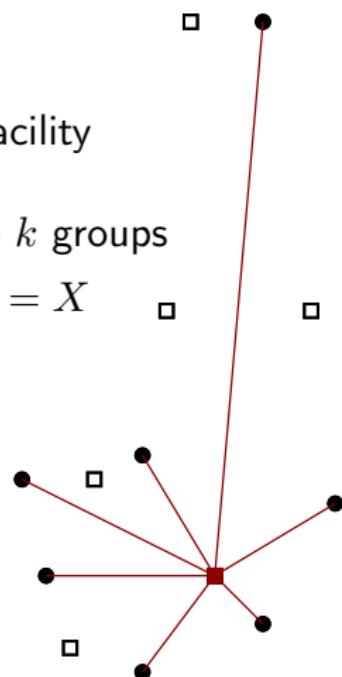
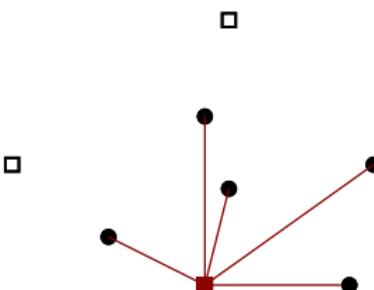
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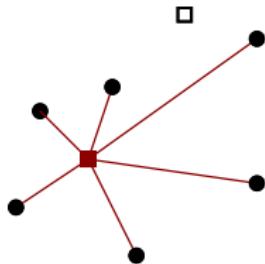
Alternative application: **cluster** objects into k groups



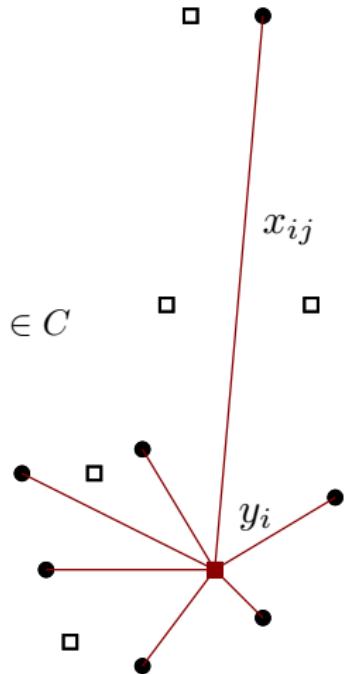
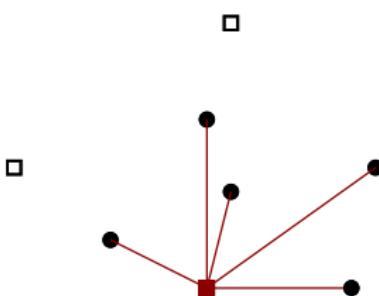
then $F = C = X$



Natural Formulation as Integer Program



$$x_{ij}, y_i \in \{0, 1\} \quad \forall i \in F, j \in C$$



Natural Formulation as Integer Program

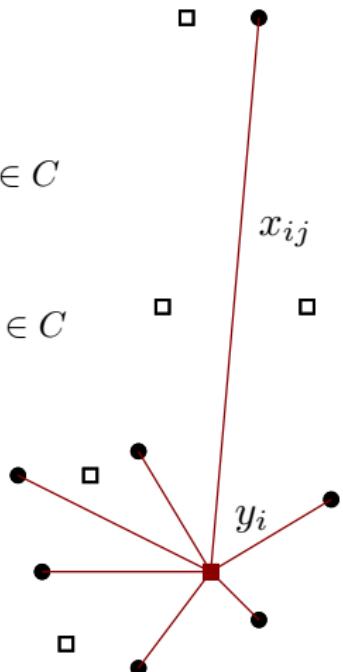
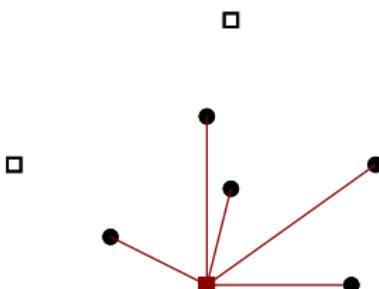
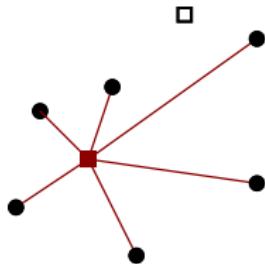
$$\min \sum_{i \in F, j \in C} c_{ij} x_{ij}, \text{ s.t.}$$

$$\sum_{i \in F} x_{ij} \geq 1 \quad \forall j \in C$$

$$x_{ij} \leq y_i \quad \forall i \in F, j \in C$$

$$\sum_{i \in F} y_i \leq k$$

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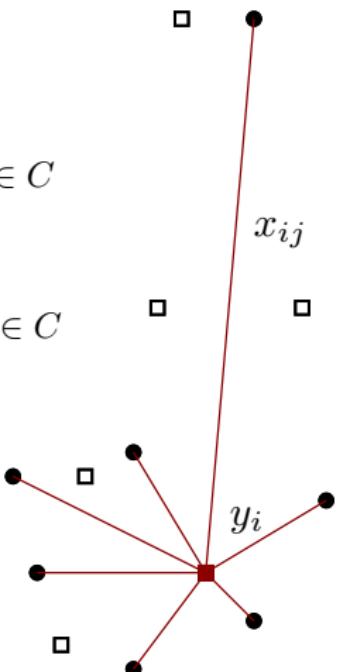
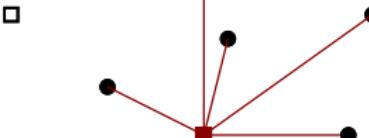
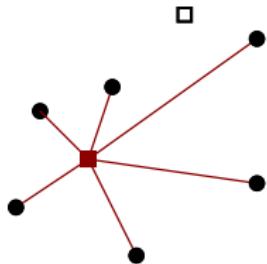
j is connected

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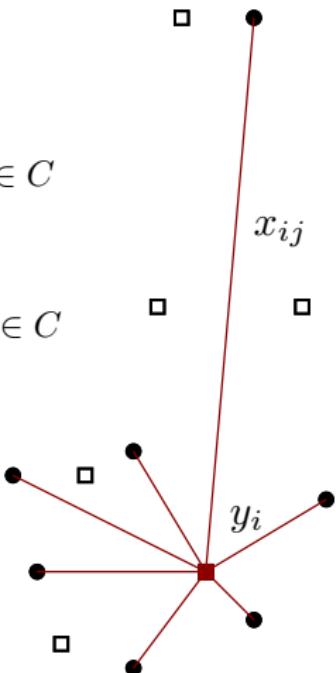
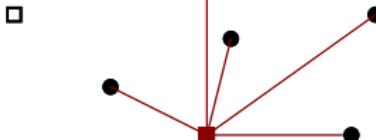
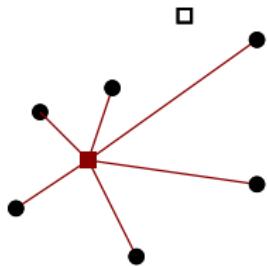
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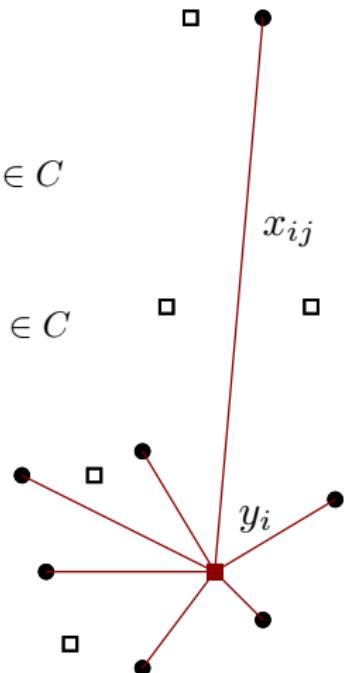
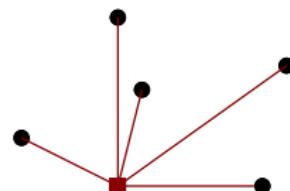
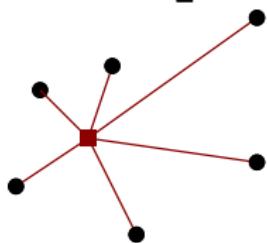
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$\leq k$ fac. are opened

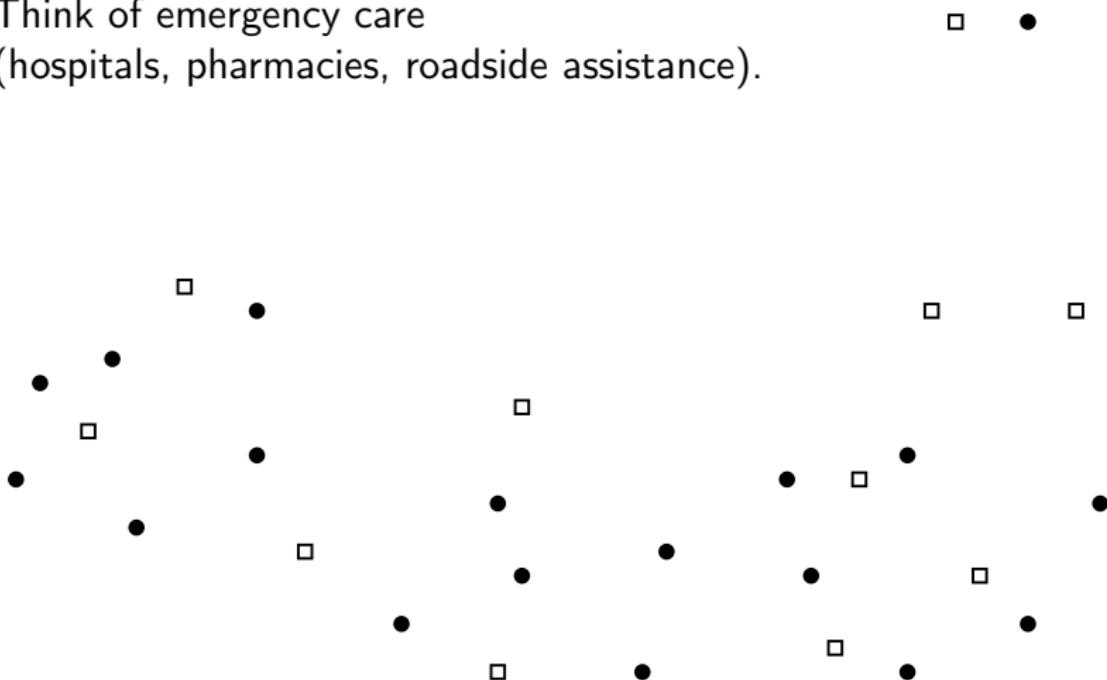
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k -CENTER Problem

Think of emergency care
(hospitals, pharmacies, roadside assistance).

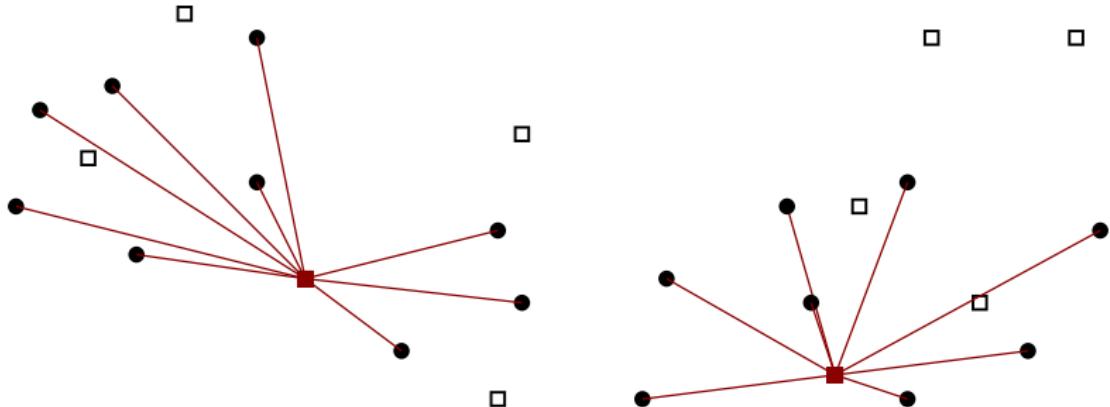


k -CENTER Problem

Think of emergency care
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minimize **highest** connection cost $\max_{j \in C} (c_{j,S})$



Unification and Interpolation

Is there a natural generalization of both models
 k -MEDIAN and k -CENTER?

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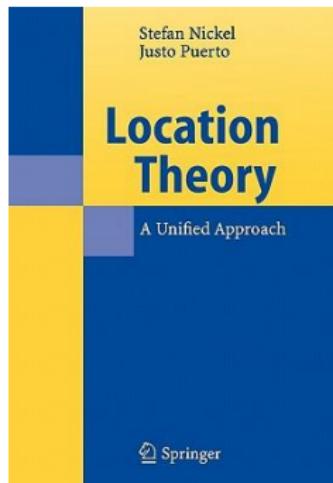
Unification and Interpolation

Is there a natural generalization of both models
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Since the 1990s there is a large body of literature in the operations research and discrete optimization communities on the ORDERED k -MEDIAN problem.



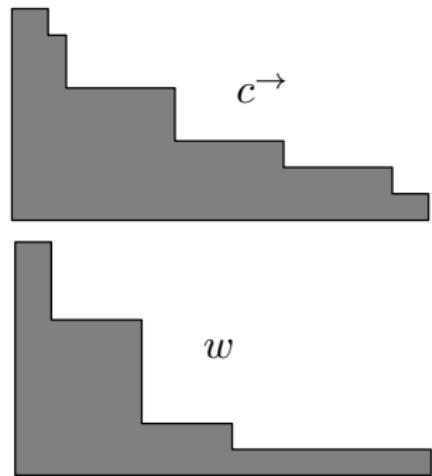
The ORDERED k -MEDIAN Model

More general objective function

For given subset S of opened facilities

$c^{\rightarrow}(S)$ — sorted connection costs

w — non-increasing weights vector



The ORDERED k -MEDIAN Model

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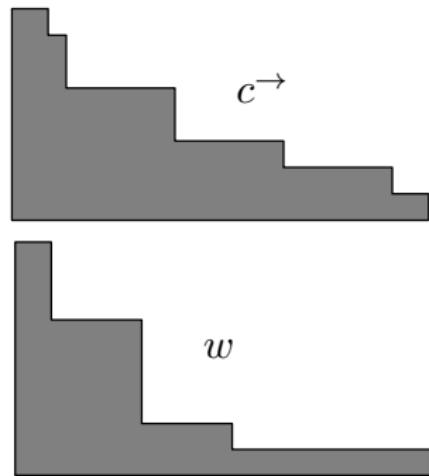
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The goal: find a k -subset $S \subseteq F$

minimizing $c^\rightarrow(S) \cdot w^\top = \sum_{j=1}^n w_j c_j^\rightarrow$



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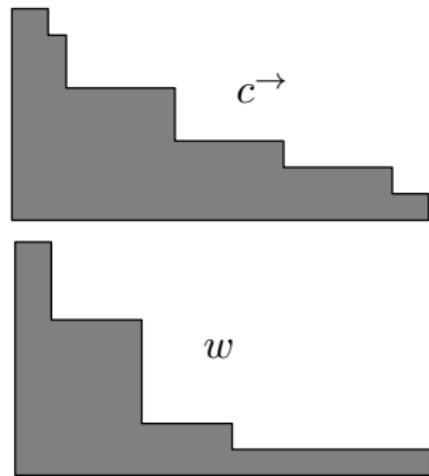
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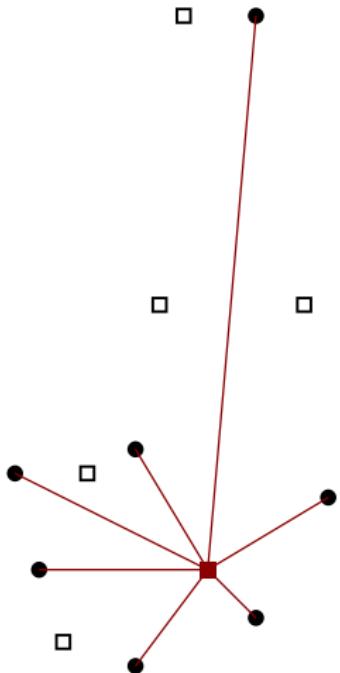
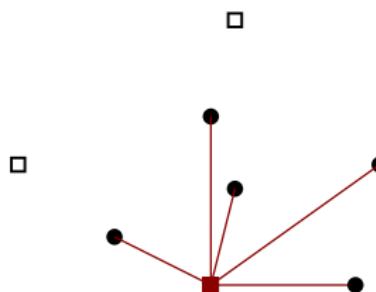
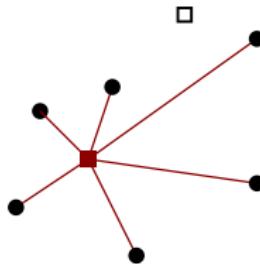
Fully flexible penalization of high connection costs



Modeling Classic k -MEDIAN

k -MEDIAN objective

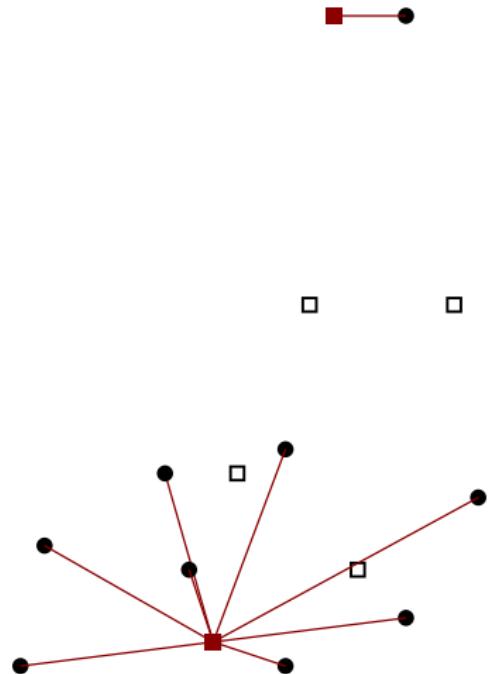
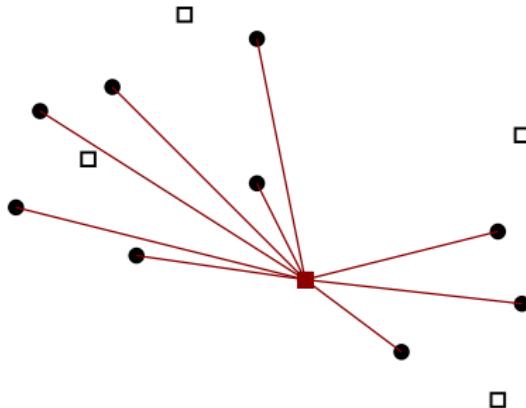
$$\text{minimize } c^\rightarrow \cdot (1, 1, \dots, 1)^\top$$



Modeling k -CENTER

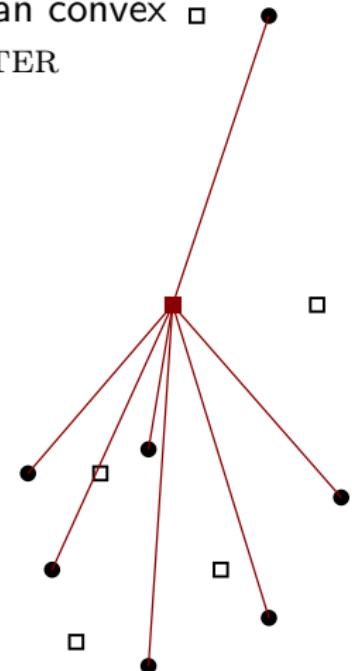
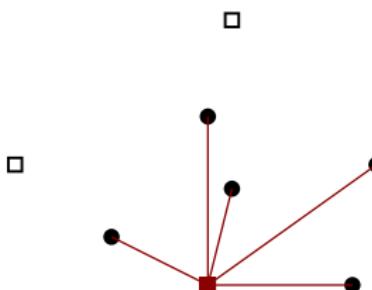
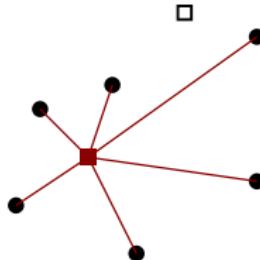
k -CENTER objective

$$\text{minimize } c^\rightarrow \cdot (1, 0, \dots, 0)^\top$$



k -CENTDIAN – Convex Combination

Consider an “intermediate” objective being a convex combination of the k -MEDIAN and k -CENTER objective



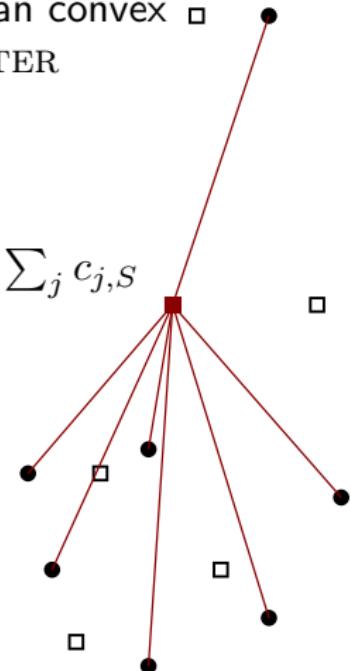
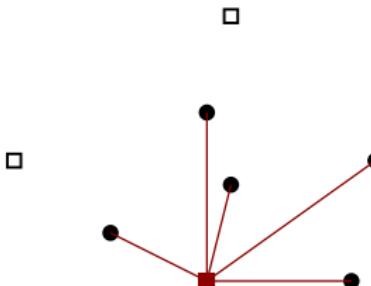
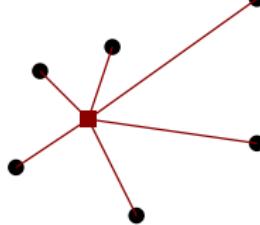
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k -CENTDIAN objective

$$\text{minimize } c^\rightarrow \cdot (1, \alpha, \dots, \alpha)^\top$$

$$= (1 - \alpha) \max_j c_{j,S} + \alpha \sum_j c_{j,S}$$



Generalization

problem	weights vector w
k -MEDIAN	(1, 1, ..., 1)
k -CENTER	(1, 0, ..., 0)
k -CENTDIAN	(1, α , ..., α)
k -FACILITY p -CENTRUM	($\underbrace{1, \dots, 1}_{p}$, 0, ..., 0)

Generalization

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k -MEDIAN and k -CENTER are NP-hard.

Approximation Algorithms

- polynomial-time in the input size
- non-optimal but close to optimal solution

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- non-optimal but close to optimal solution

$$\text{cost(ALG)} \leq \alpha \cdot \text{cost(OPT)}$$

α is called an approximation ratio/factor

Approximation Algorithms for OkM

Problem	Factor	Technique	Authors	Year
k -MEDIAN	$6\frac{1}{2}$	LP rounding	Charikar et al.	1999
k -MEDIAN	6	primal dual	Jain, Vazirani	1999
k -MEDIAN	4	primal dual	Jain et al.	2002
k -MEDIAN	$3 + \epsilon$	local search	Arya et al.	2001
k -MEDIAN	2.732	primal dual	Li, Svensson	2013
k -MEDIAN	2.675	primal dual	Byrka et. al.	2015
k -CENTER	2 (tight)	greedy	Hochbaum, Shmoys	1985
OkM	$O(\log n)$	local search	Aouad, Segev	2017

Difficulties of OkM: non-linear, ranking-based objective function

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Constant-factor approximation for ORDERED k -MEDIAN?

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Constant-factor approximation for ORDERED k -MEDIAN?

An open question even for the special case of
 k -FACILITY p -CENTRUM problem, $w = (1, 1, \dots, 1, 0, \dots, 0)$
 (Tamir'01, Alamdari, Shmoys'17, Aouad, Segev'17)

Main Result

Theorem

There is a constant-factor approximation algorithm for ORDERED k -MEDIAN that runs in poly-time.

LP-based Approximation Algorithms

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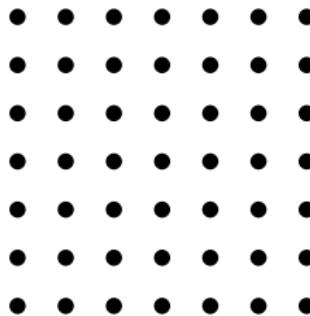
integer programming



LP-based Approximation Algorithms

integer programming

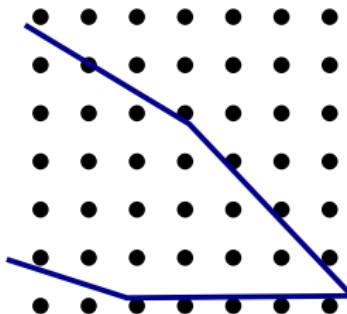
$$x_i \in \{0, 1\}$$



LP-based Approximation Algorithms

integer programming

$$\begin{aligned}x_i &\in \{0, 1\} \\Ax &\leq b\end{aligned}$$



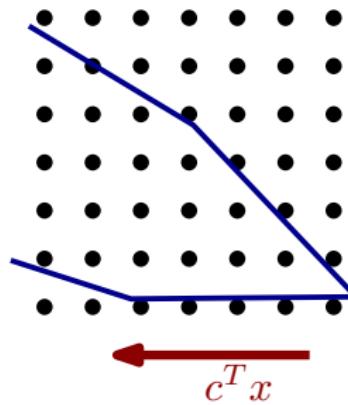
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$$Ax \leq b$$

$$\text{minimize } c^T x$$



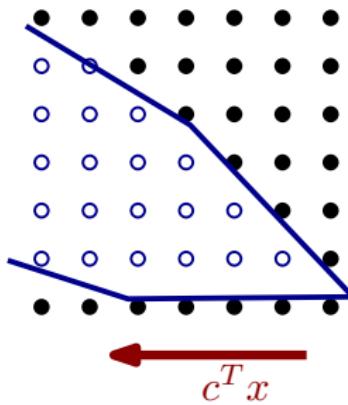
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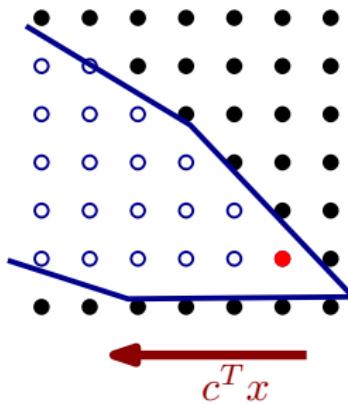
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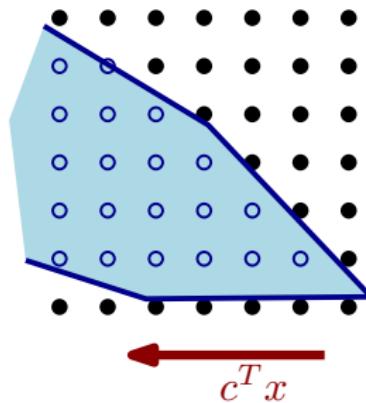
$$\text{minimize } c^T x$$

linear programming

$$x_i \in [0, 1]$$

$$Ax \leq b$$

$$\text{minimize } c^T x$$



LP-based Approximation Algorithms

integer programming

$$x_i \in \{0, 1\}$$

$$Ax \leq b$$

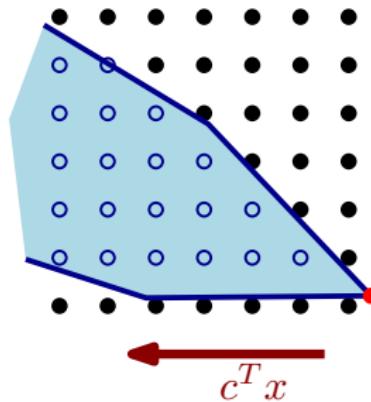
$$\text{minimize } c^T x$$

linear programming

$$x_i \in [0, 1]$$

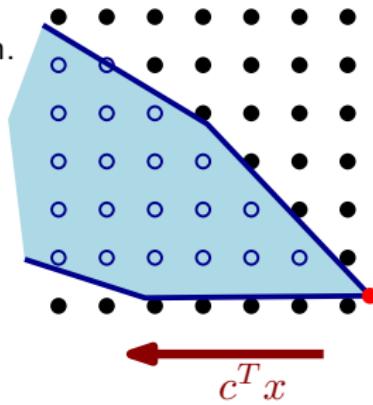
$$Ax \leq b$$

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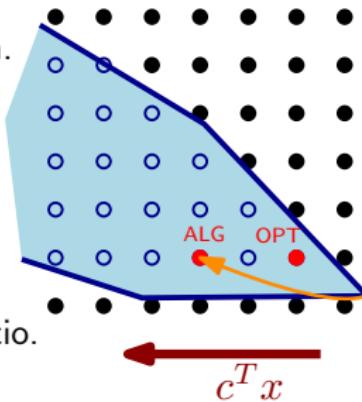
LP-based Approximation Algorithms

1. Define problem as an integer program.
2. Define proper linear program.



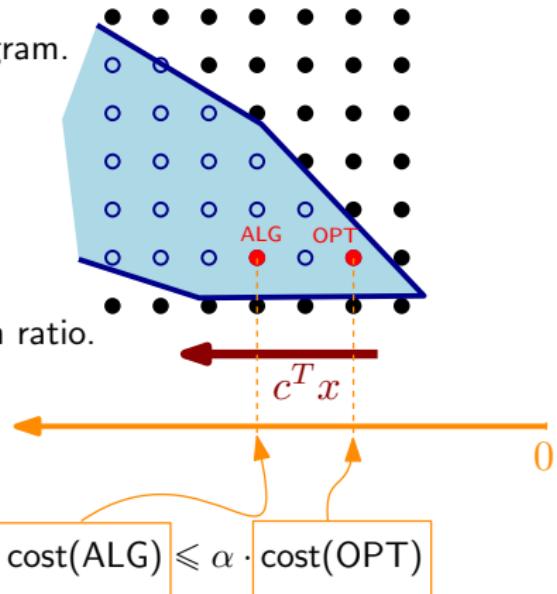
LP-based Approximation Algorithms

1. Define problem as an integer program.
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3. Define a rounding procedure.
4. Prove best possible approximation ratio.



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Main Result

Theorem

There is a constant-factor approximation algorithm for ORDERED k -MEDIAN that runs in poly-time.

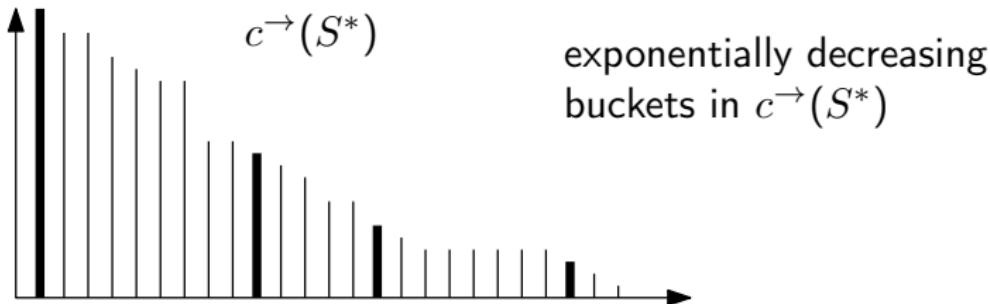
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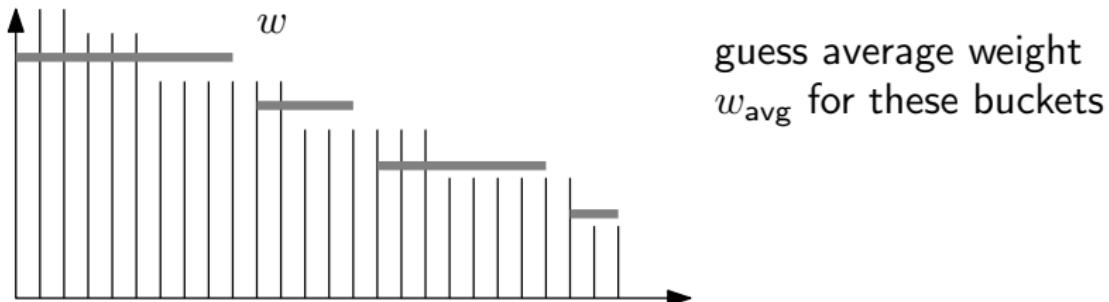
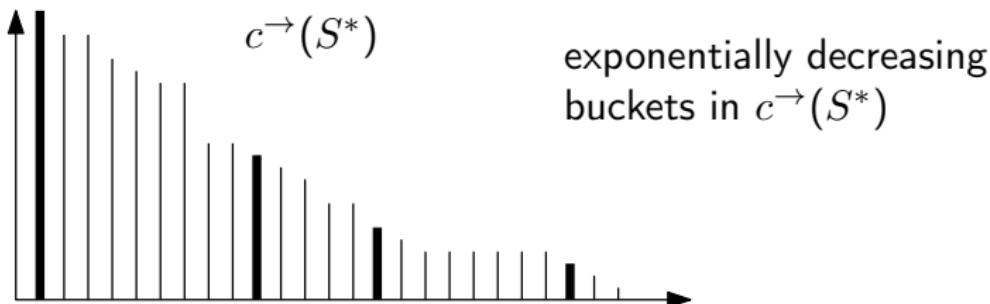
There is a constant-factor approximation algorithm for ORDERED k -MEDIAN that runs in poly-time.

transform metric cost c into
non-metric cost c^r

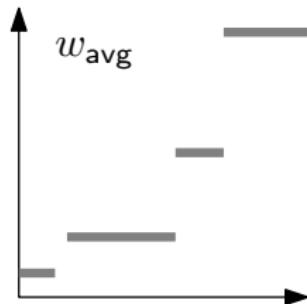
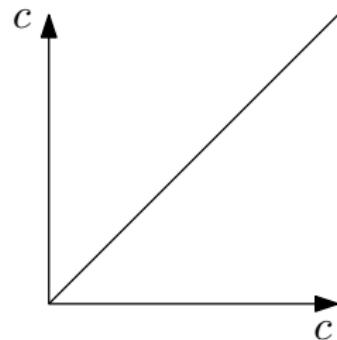
Distance Bucketing w. Average Weights



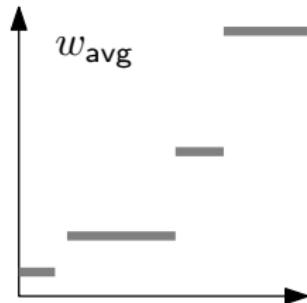
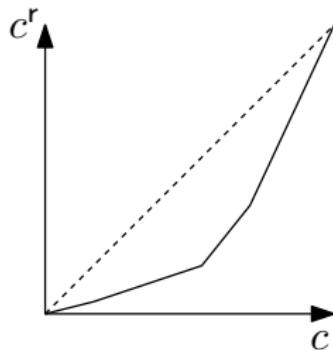
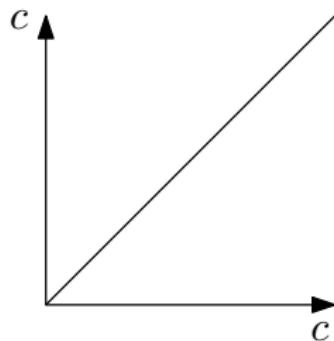
Distance Bucketing w. Average Weights



(Non-Metric) Reduced Cost-Space



(Non-Metric) Reduced Cost-Space



apply average weight vector
to metric space

Main Result

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Main Result

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There is a constant-factor approximation algorithm for ORDERED k -MEDIAN that runs in poly-time.

transform metric cost c into
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solve k -MEDIAN LP relaxation
over new **non-metric** cost c^r

Relaxation over Distorted Cost-Space

$$\min \sum_{j \in C} \sum_{i \in F} c_{ij}^r x_{ij}, \quad \text{s.t.}$$

$$\sum_{i \in F} x_{ij} \geq 1 \quad \forall j \in C$$

$$x_{ij} \leq y_i \quad \forall i \in F, j \in C$$

$$\sum_{i \in F} y_i \leq k$$

$$x_{ij}, y_i \geq 0 \quad \forall i \in F, j \in C$$

Main Result

Theorem

There is a constant-factor approximation algorithm for ORDERED k -MEDIAN that runs in poly-time.

transform metric cost c into
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solve k -MEDIAN LP relaxation
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Main Result

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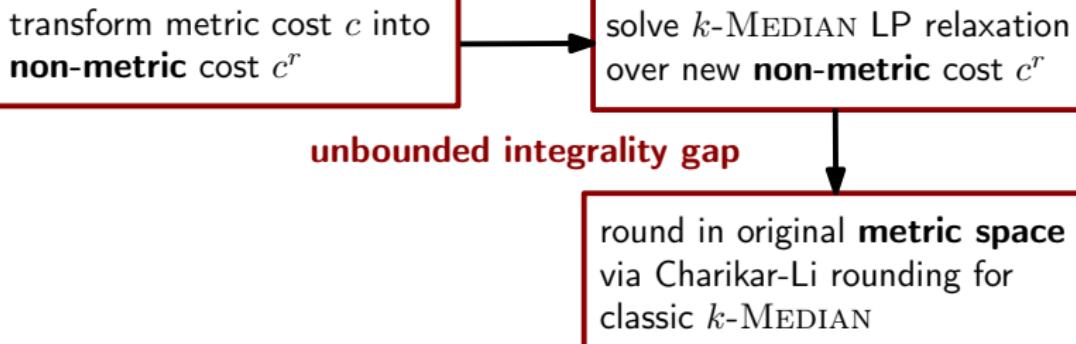
solve k -MEDIAN LP relaxation
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unbounded integrality gap

Main Result

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Relaxation over Distorted Cost-Space

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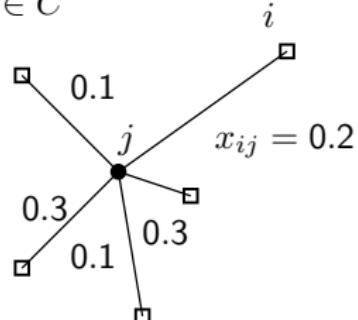
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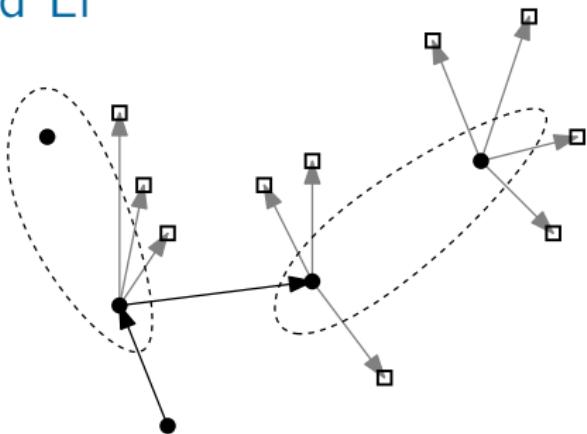
$$c_{\text{av}}^r(j) := \boxed{\sum_{i \in F} c_{ij}^r x_{ij}}$$

$$c_{\text{av}}(j) := \sum_{i \in F} c_{ij} x_{ij}$$



Analysis of Charikar and Li

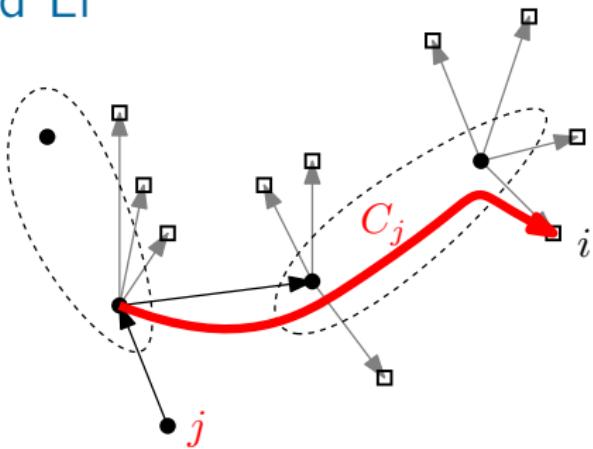
original analysis of
Charikar and Li [ICALP 2012]



Analysis of Charikar and Li

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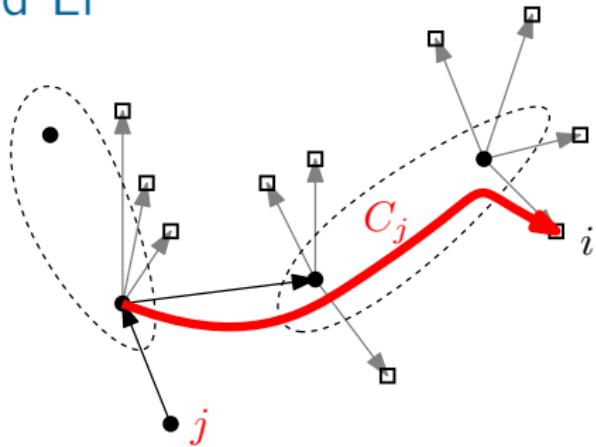
random connection cost C_j of
any client j fulfill
 $\mathbb{E}[C_j] = O(c_{av}(j))$



Analysis of Charikar and Li

original analysis of
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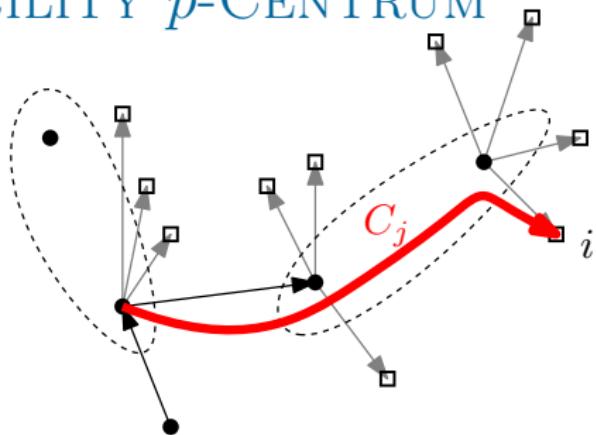
random connection cost C_j of
any client j fulfill
 $\mathbb{E}[C_j] = O(c_{\text{av}}(j))$



$$\begin{aligned}\mathbb{E}[c(A)] &= \mathbb{E} \left[\sum_j C_j \right] = \sum_j \mathbb{E}[C_j] \\ &= O \left(\sum_j c_{\text{av}}(j) \right) = O(\text{OPT})\end{aligned}$$

Idea of Analysis k -FACILITY p -CENTRUM

difficulty for
 k -FACILITY p -CENTRUM:
non-linearity of objective



Idea of Analysis k -FACILITY p -CENTRUM

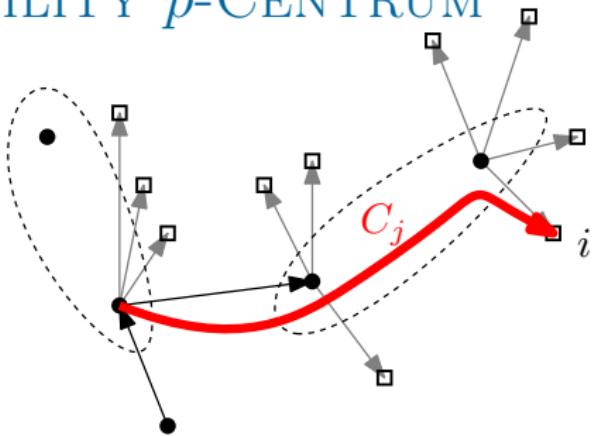
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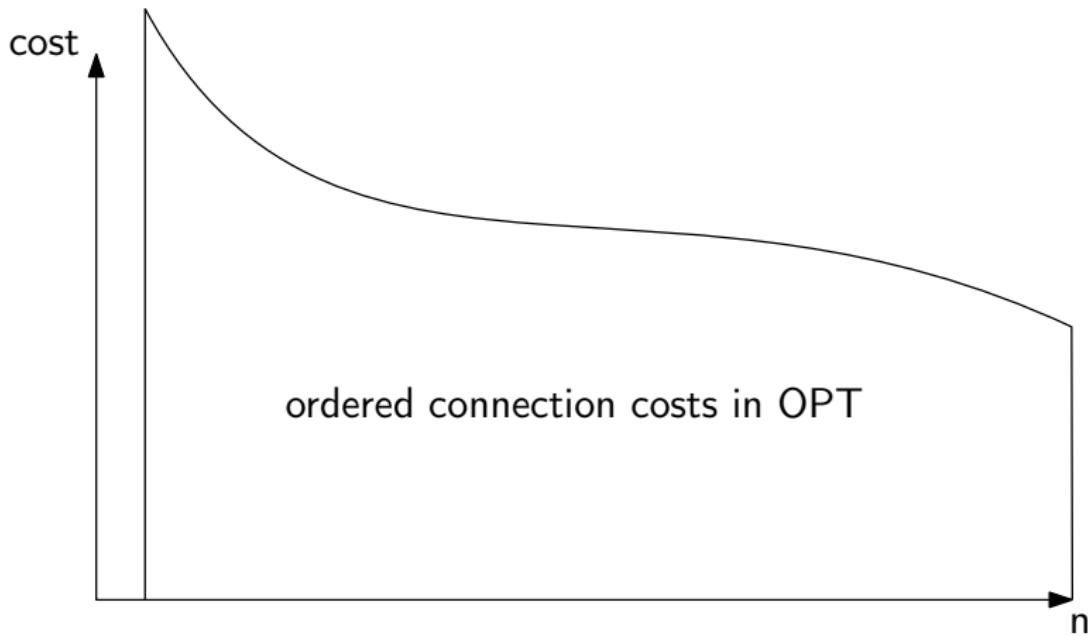
non-linearity of objective

even for $p = 1$, we could have

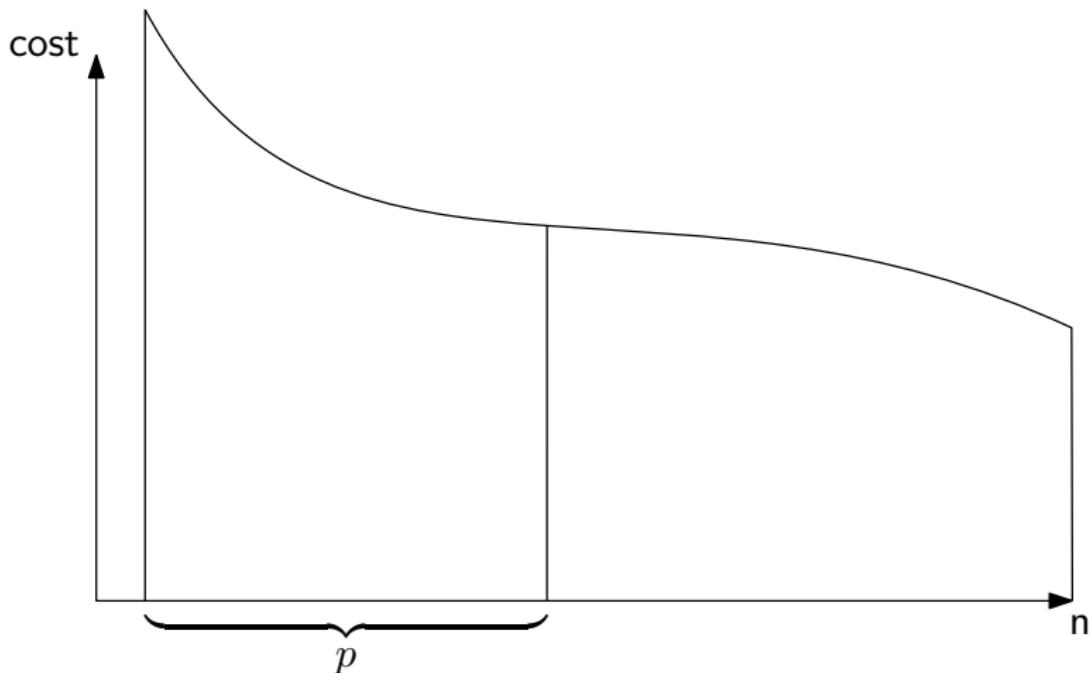
$\mathbb{E}[\max_j C_j] \gg \max_j \mathbb{E}[C_j]$



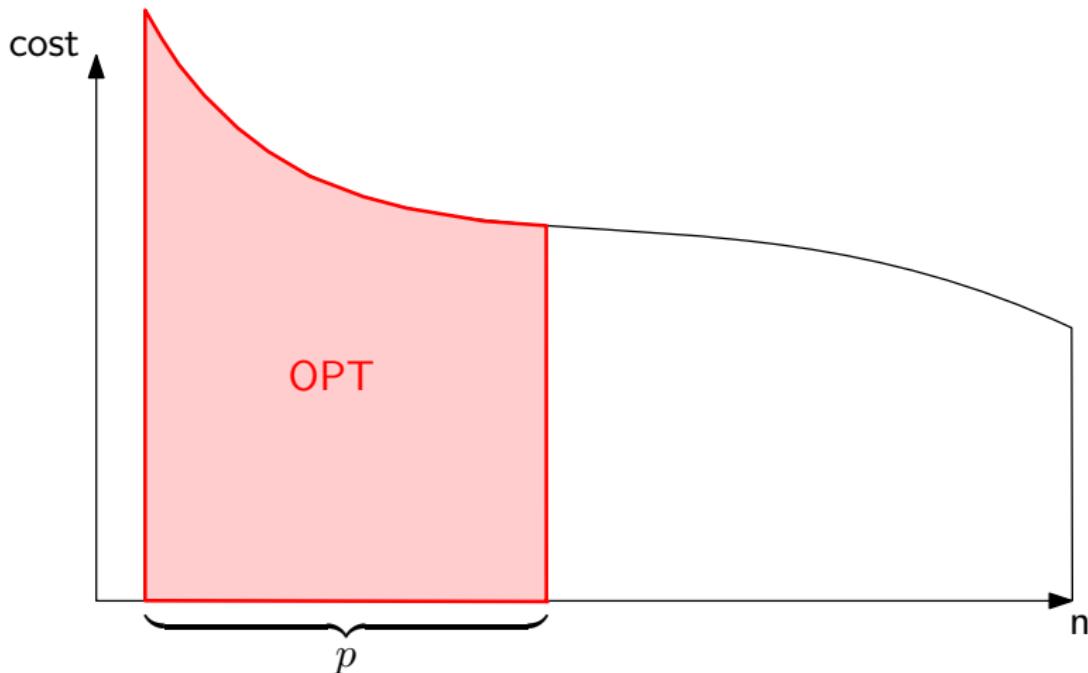
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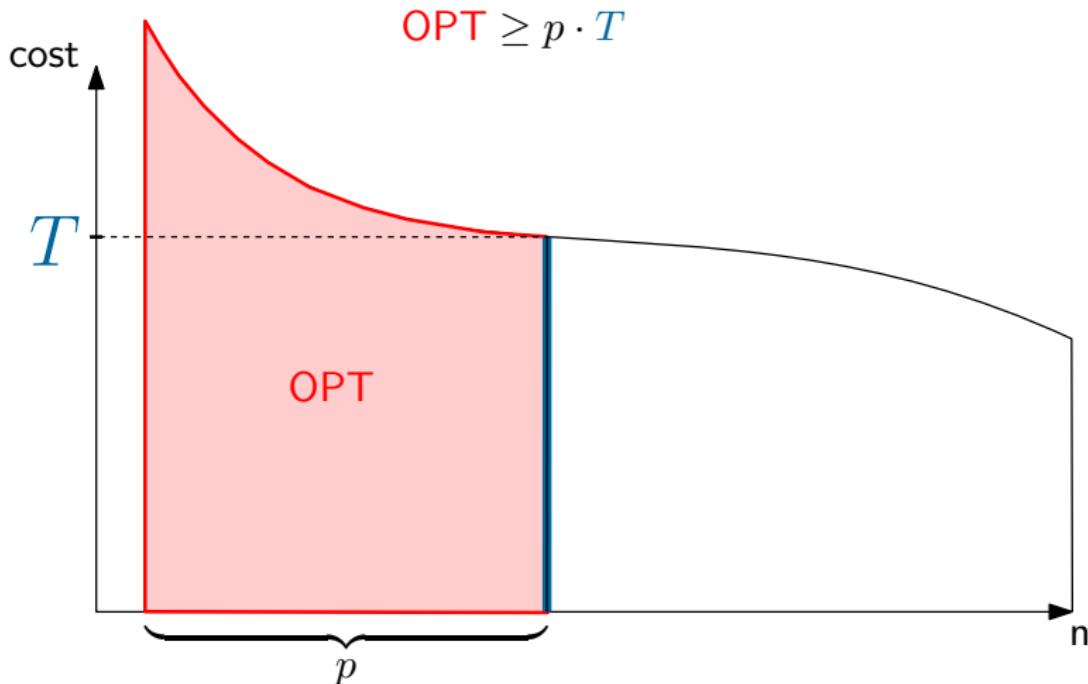
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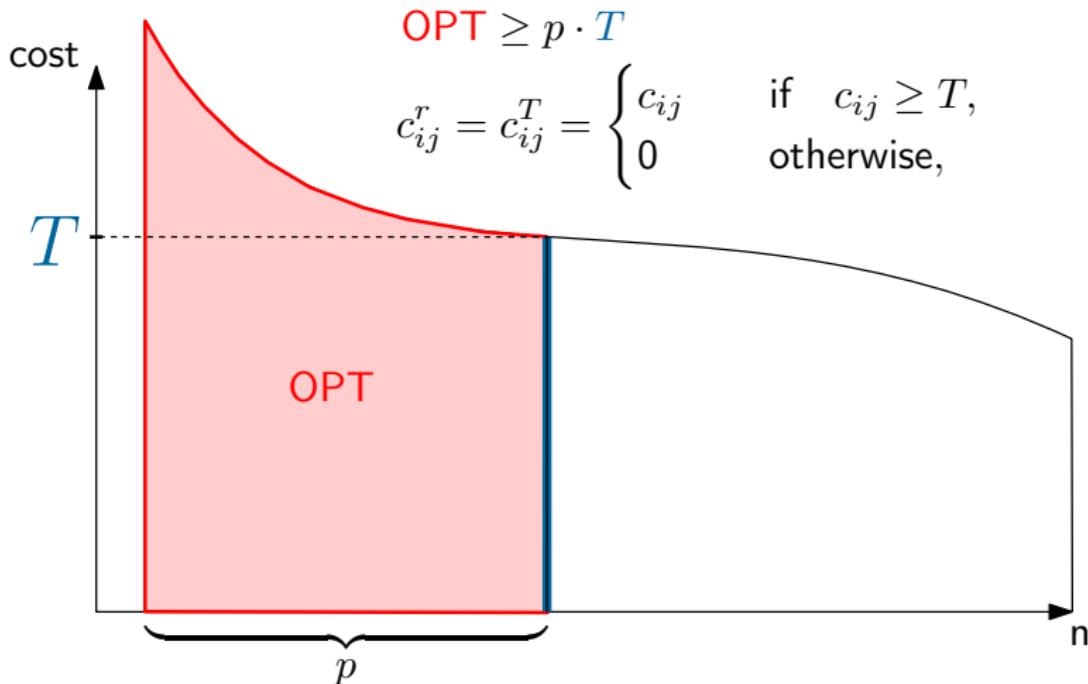
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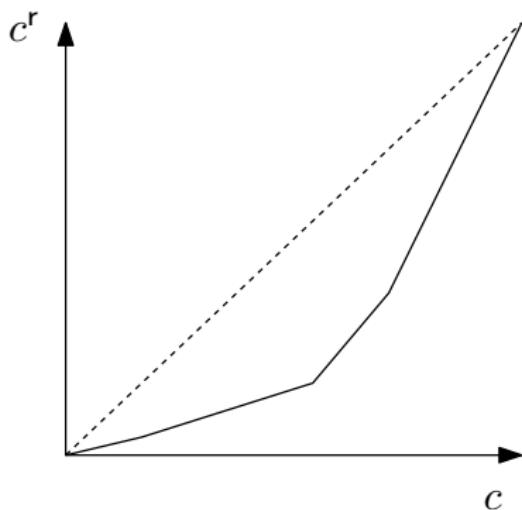
Idea of Analysis k -FACILITY p -CENTRUM



Idea of Analysis k -FACILITY p -CENTRUM

$$\text{OPT} \geq p \cdot T$$

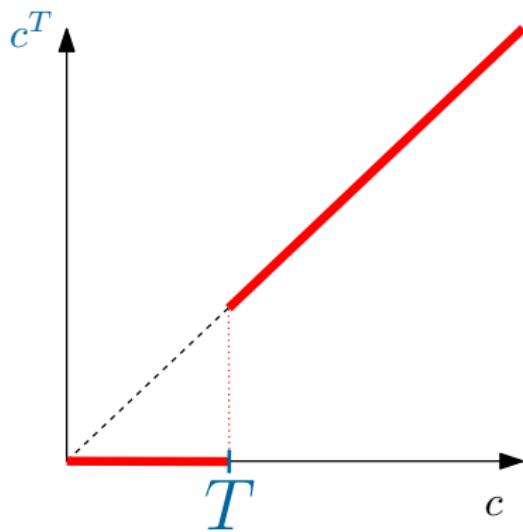
$$c_{ij}^r = c_{ij}^T = \begin{cases} c_{ij} & \text{if } c_{ij} \geq T, \\ 0 & \text{otherwise,} \end{cases}$$



Idea of Analysis k -FACILITY p -CENTRUM

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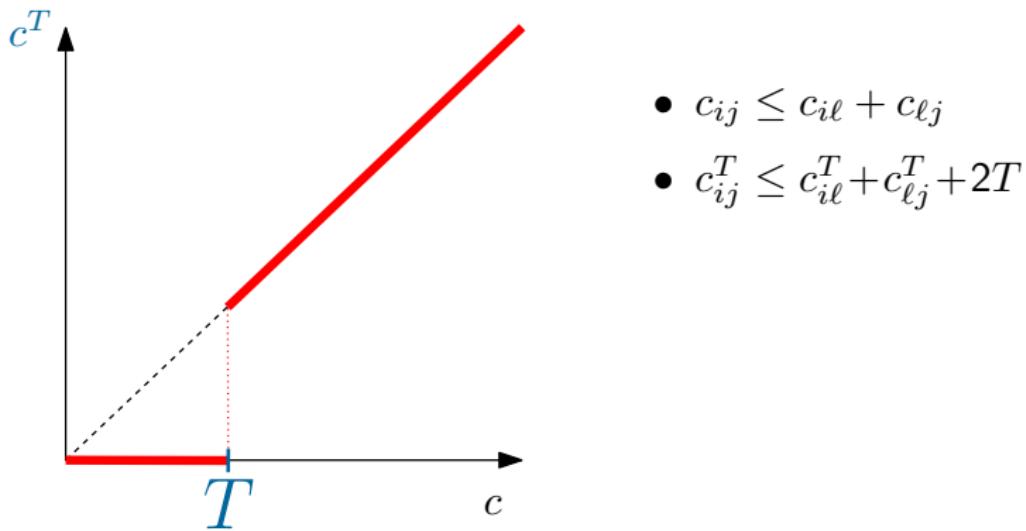
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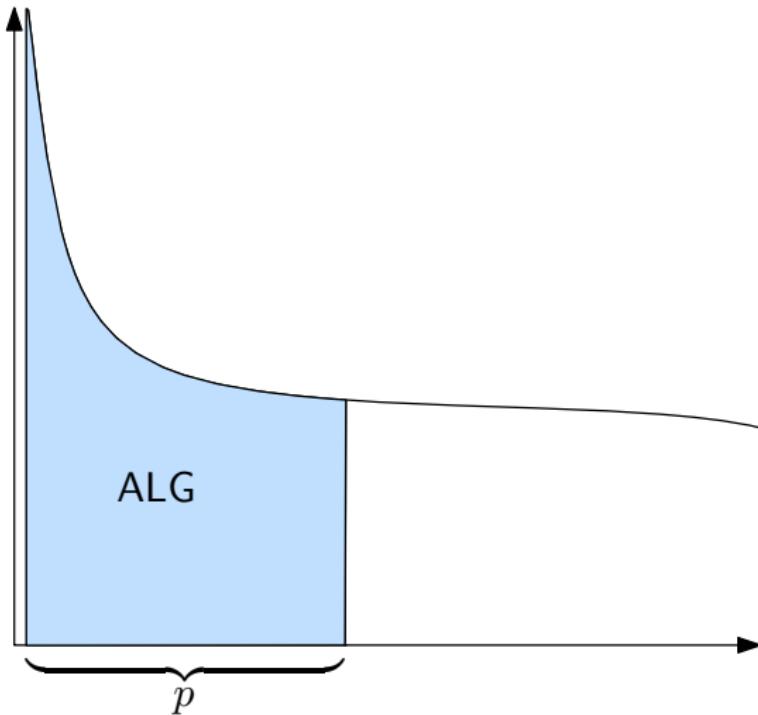
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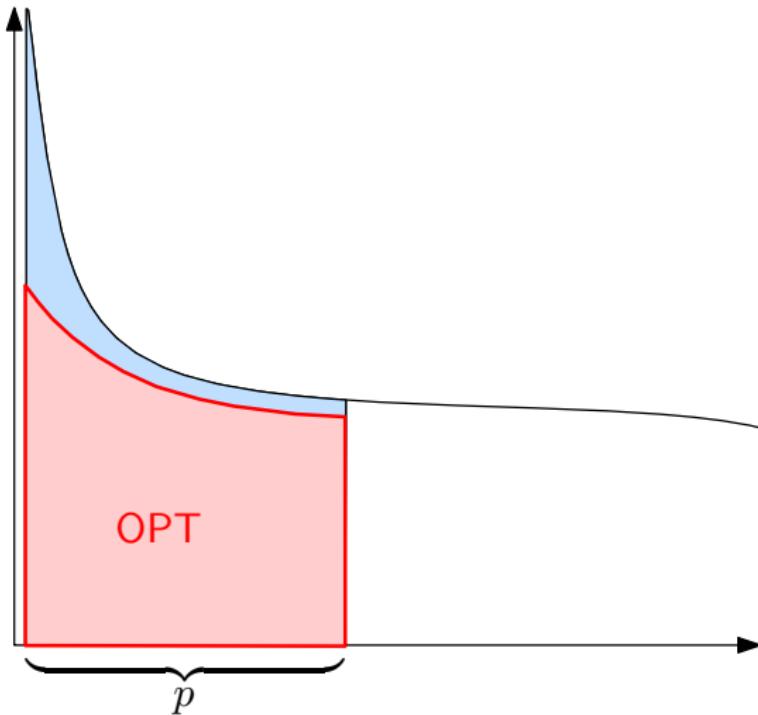
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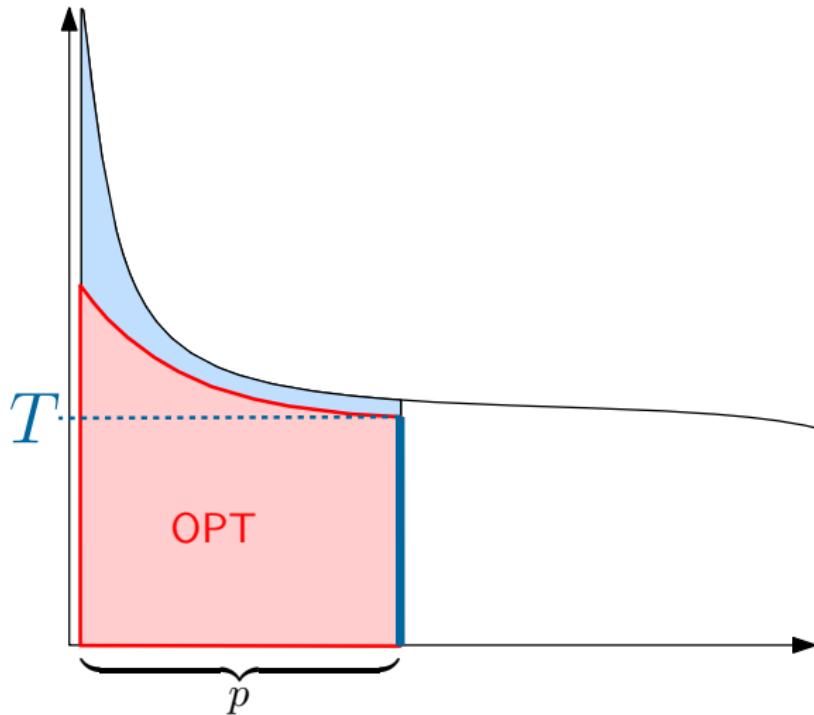
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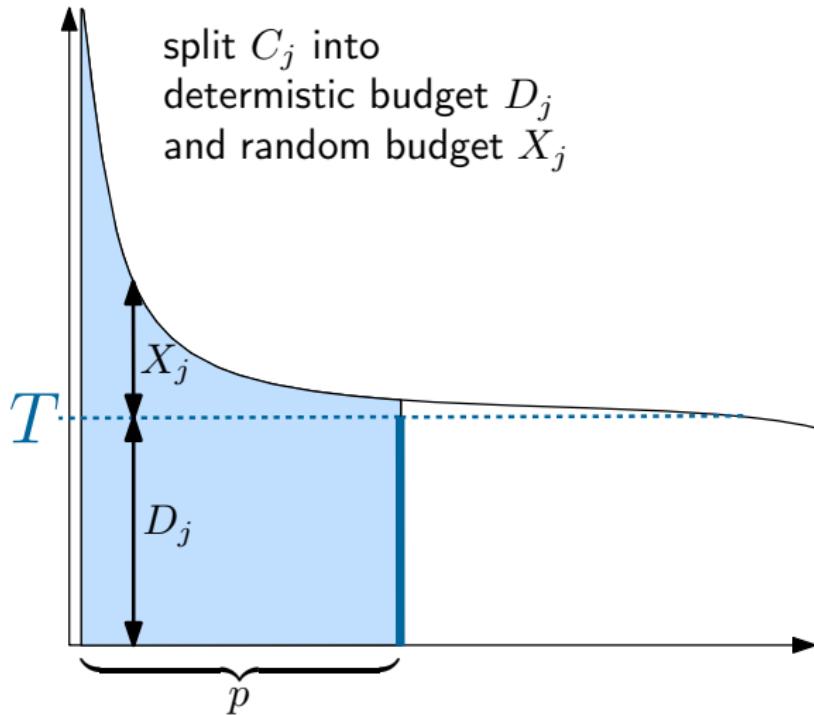
Idea of Analysis k -FACILITY p -CENTRUM



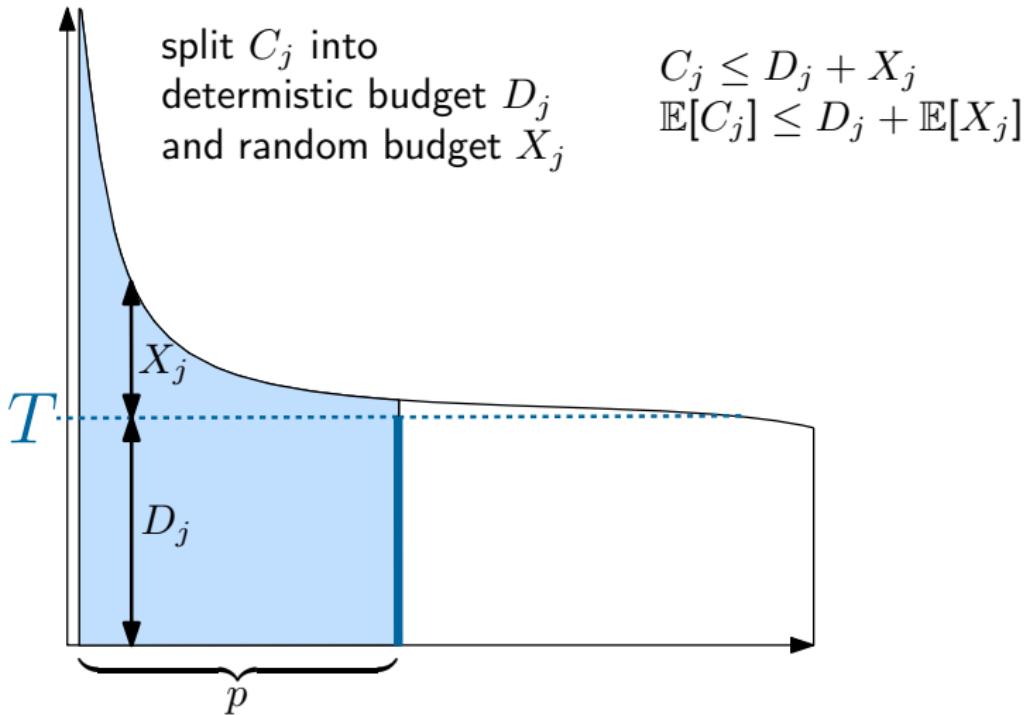
Idea of Analysis k -FACILITY p -CENTRUM



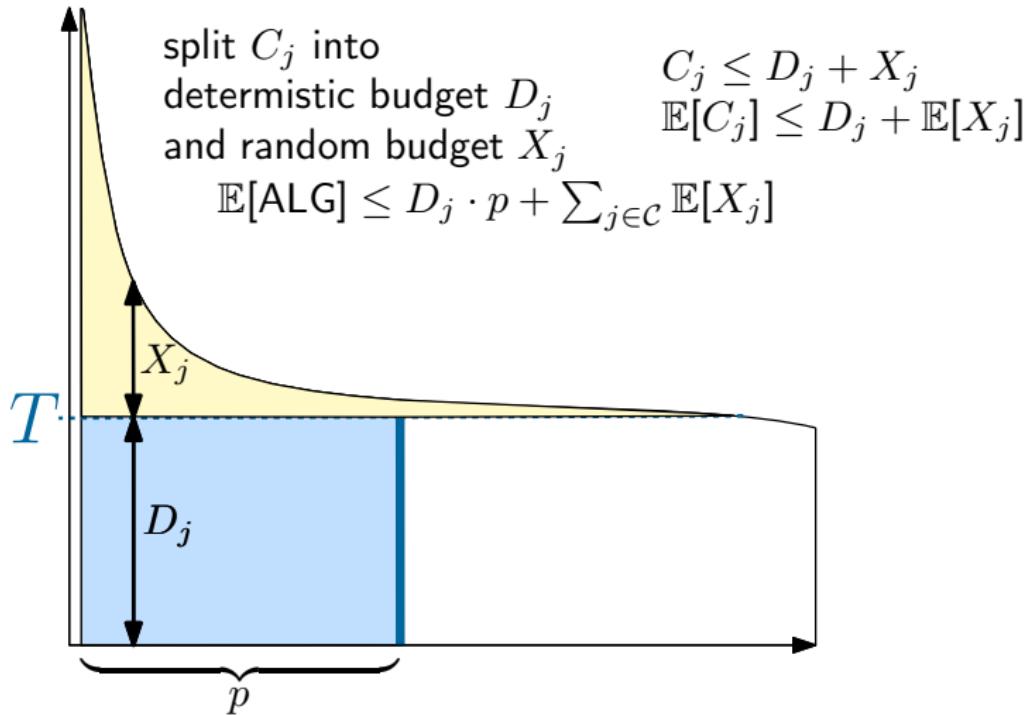
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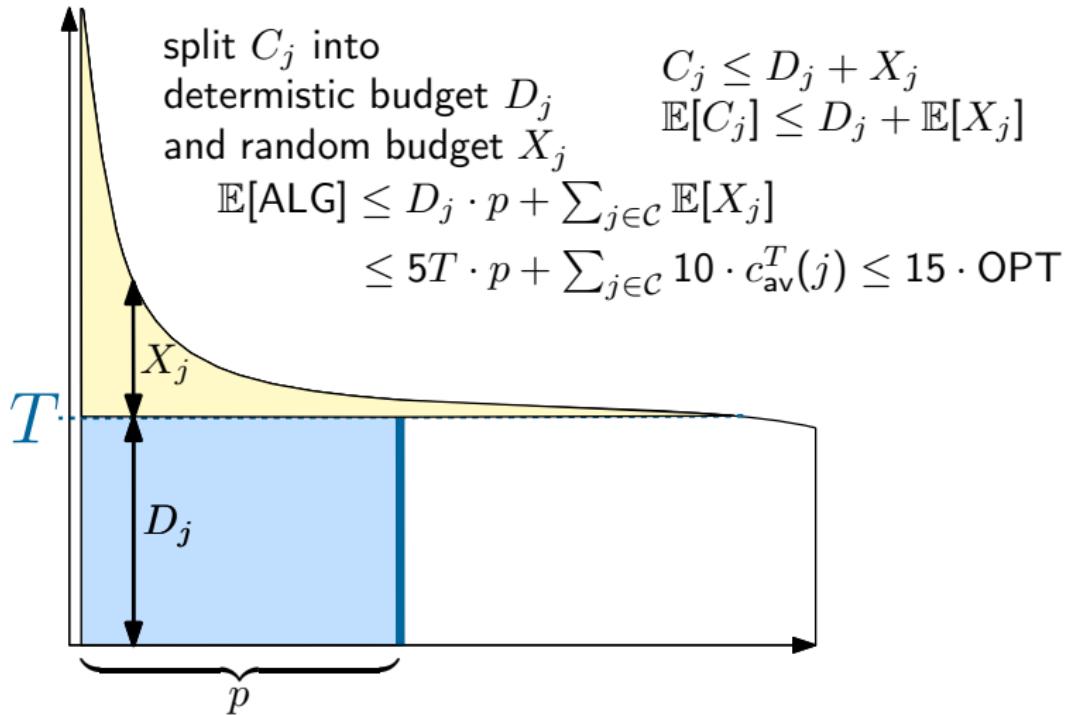
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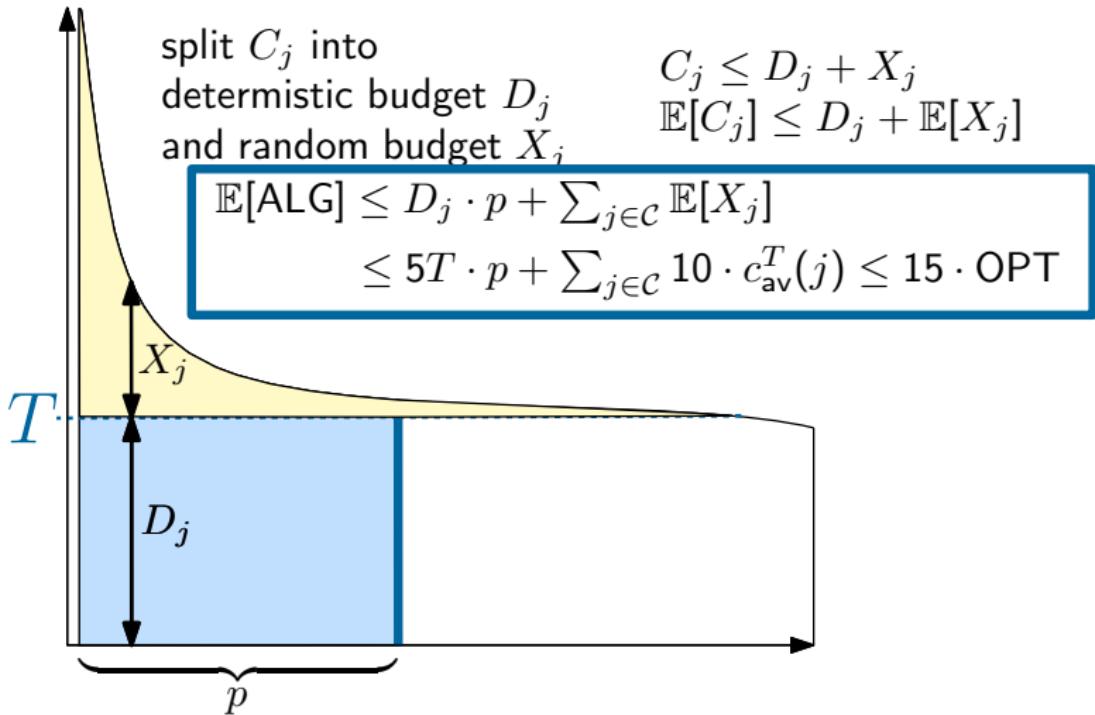
Idea of Analysis k -FACILITY p -CENTRUM



Idea of Analysis k -FACILITY p -CENTRUM



Idea of Analysis k -FACILITY p -CENTRUM



General case of ORDERED k -MEDIAN

Dedicated algorithm for k -FACILITY p -CENTRUM

$$\begin{aligned}\mathbb{E}[\text{ALG}] &\leq D_j \cdot p + \sum_{j \in \mathcal{C}} \mathbb{E}[X_j] \\ &\leq 5T \cdot p + \sum_{j \in \mathcal{C}} 10 \cdot c_{\text{av}}^T(j) \leq 15 \cdot \text{OPT}\end{aligned}$$

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Oblivious algorithm for k -FACILITY p -CENTRUM

$$\mathbb{E}[\text{cost}_p(\text{ALG})] \leq 19T \cdot p + 19 \sum_{j \in C} c_{\text{av}}^T(j)$$

General case of ORDERED k -MEDIAN

Dedicated algorithm for k -FACILITY p -CENTRUM

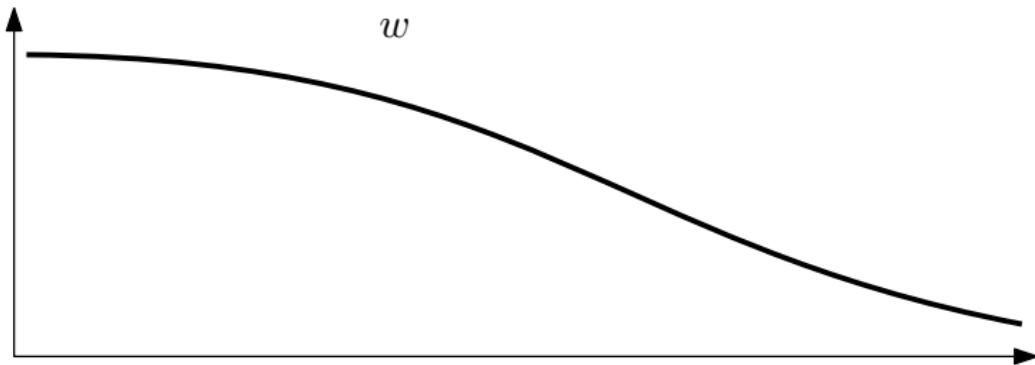
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Oblivious algorithm for k -FACILITY p -CENTRUM

$$\forall T \geq 0, p \geq 1 \quad \mathbb{E}[\text{cost}_p(\text{ALG})] \leq 19T \cdot p + 19 \sum_{j \in C} c_{\text{av}}^T(j)$$

modularity lemma

General case of ORDERED k -MEDIAN

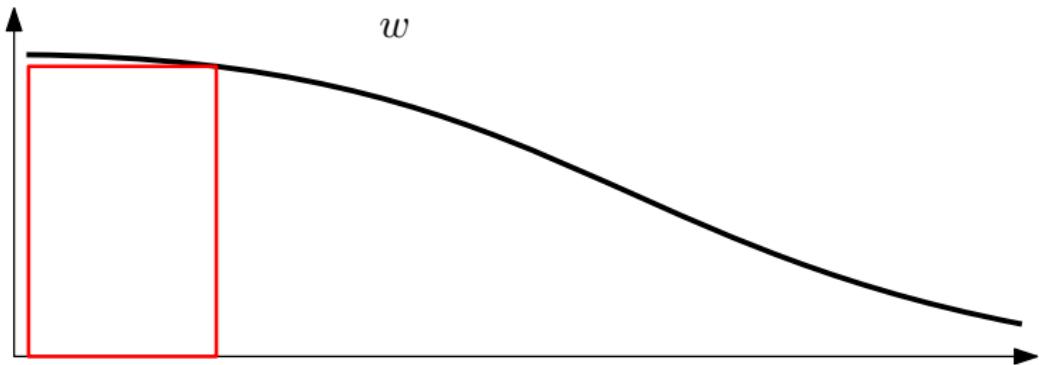


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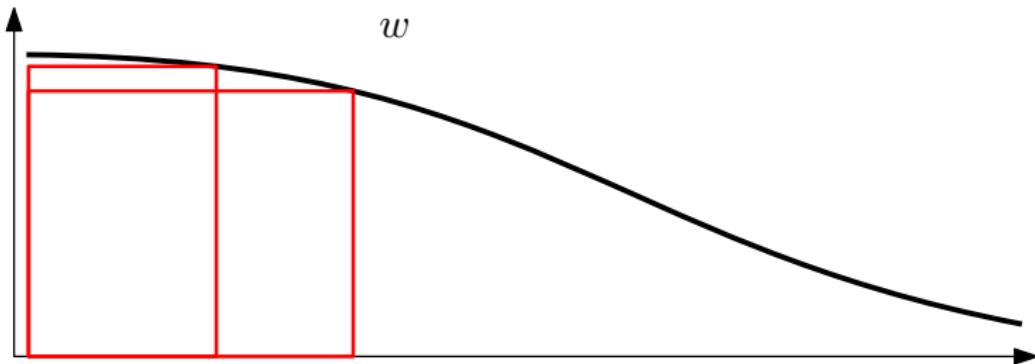


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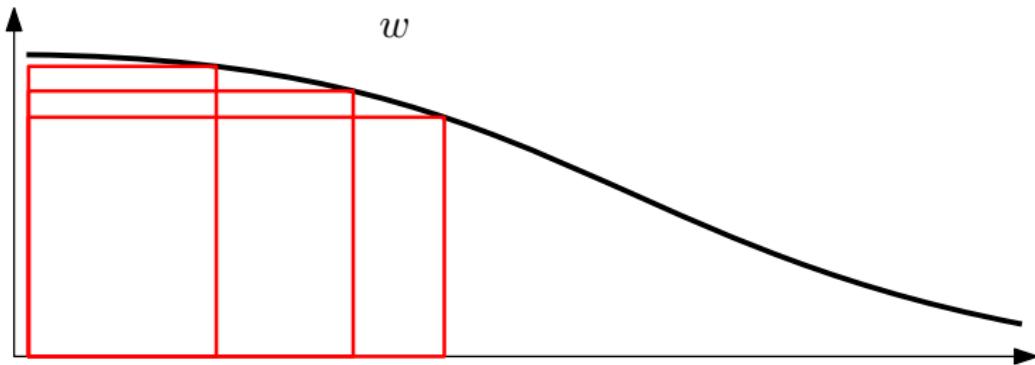


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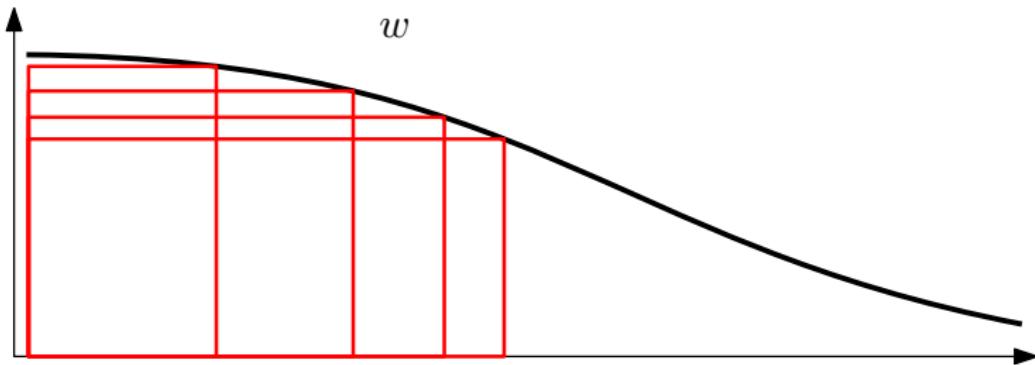


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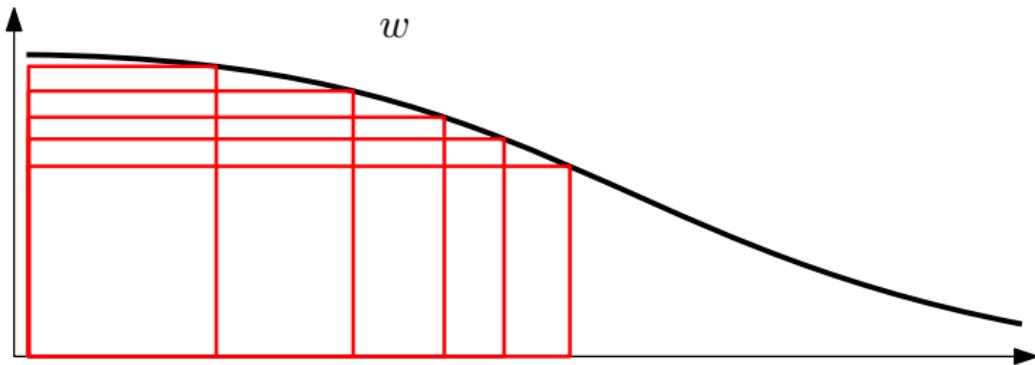


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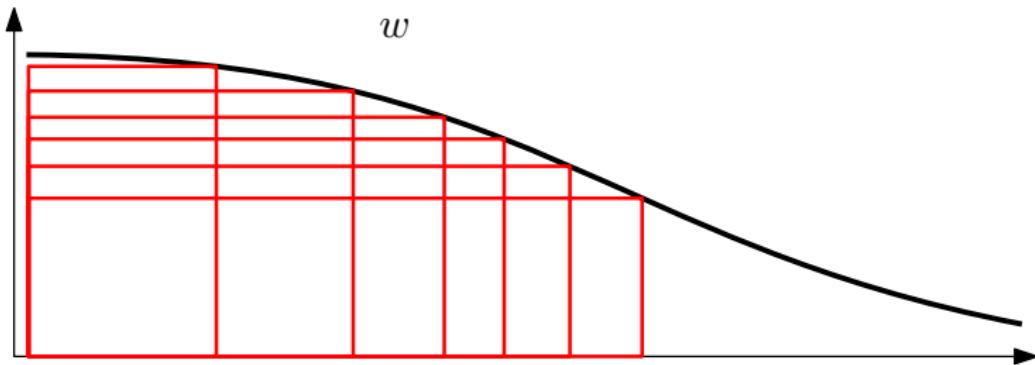


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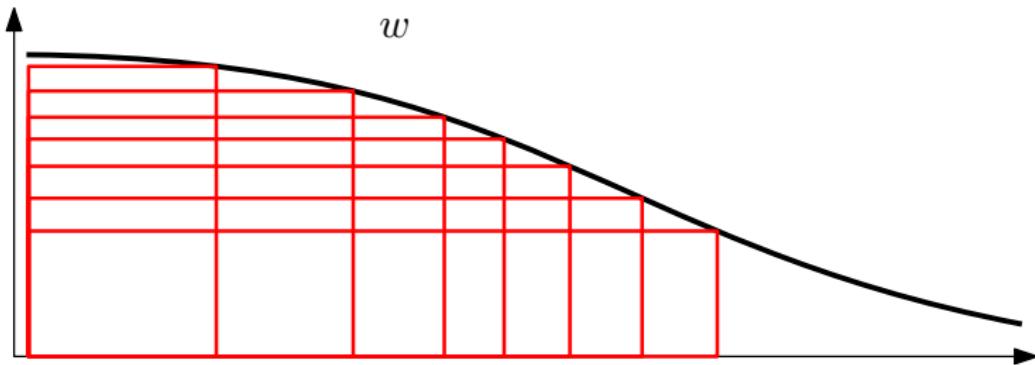


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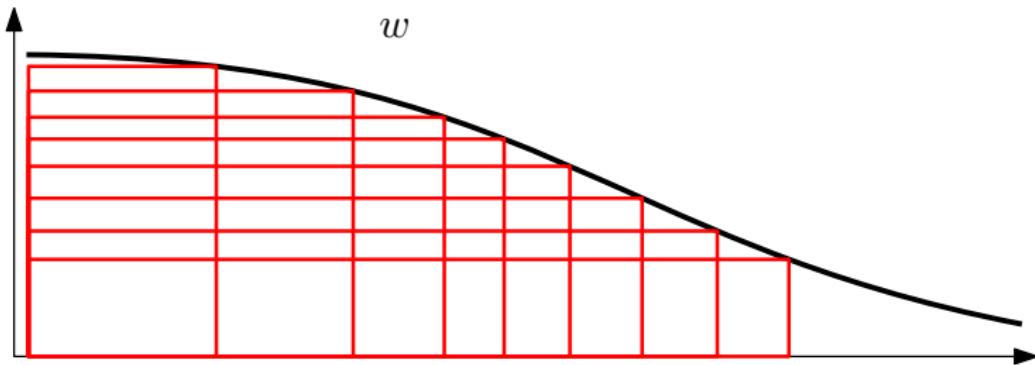


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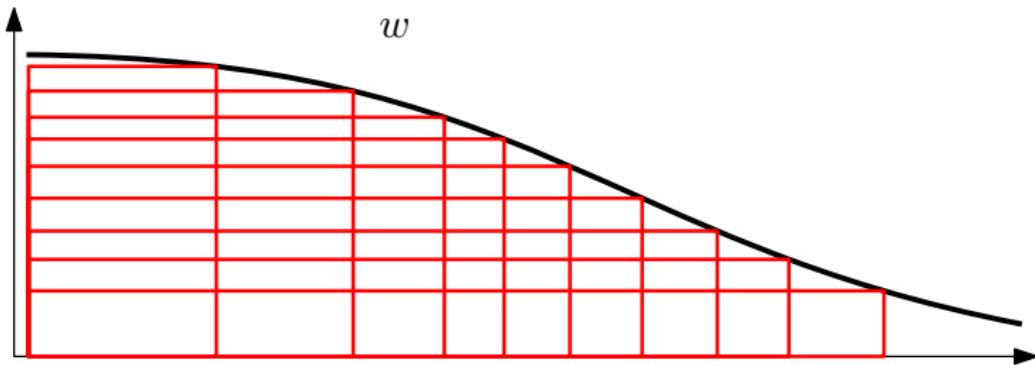


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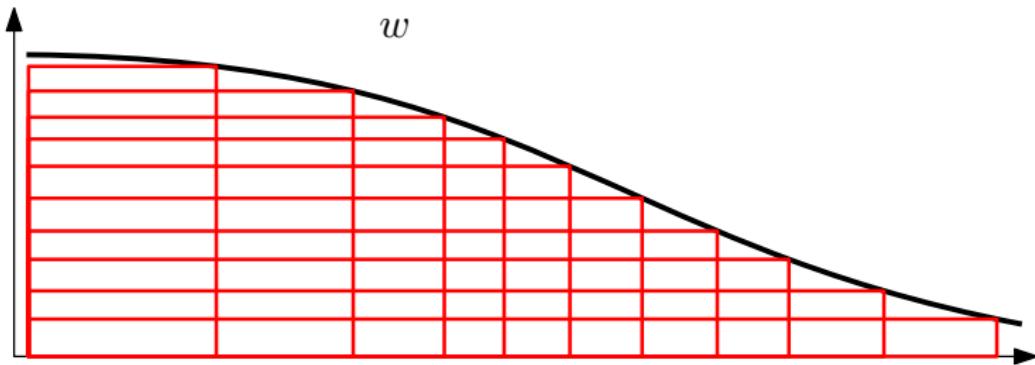


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modularity lemma

Approximation Algorithms for OkM

Problem	Factor	Technique	Authors	Year
k -MEDIAN	$6\frac{1}{2}$	LP rounding	Charikar et al.	1999
k -MEDIAN	6	primal dual	Jain, Vazirani	1999
k -MEDIAN	4	primal dual	Jain et al.	2002
k -MEDIAN	$3 + \epsilon$	local search	Arya et al.	2001
k -MEDIAN	2.732	primal dual	Li, Svensson	2013
k -MEDIAN	2.675	primal dual	Byrka et. al.	2015
k -CENTER	2 (tight)	greedy	Hochbaum, Shmoys	1985
OkM	$O(\log n)$	local search	Aouad, Segev	2017

Constant-factor approximation for ORDERED k -MEDIAN?

An open question even for the special case of
 k -FACILITY p -CENTRUM problem, $w = (1, 1, \dots, 1, 0, \dots, 0)$

Approximation Algorithms for OkM

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OkM	$18 + \epsilon$	primal-dual	Chakrabarty et al.	ICALP'18

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k -MEDIAN	$6\frac{1}{2}$	LP rounding	Charikar et al.	1999
k -MEDIAN	6	primal dual	Jain, Vazirani	1999
k -MEDIAN	4	primal dual	Jain et al.	2002
k -MEDIAN	$3 + \epsilon$	local search	Arya et al.	2001
k -MEDIAN	2.732	primal dual	Li, Svensson	2013
k -MEDIAN	2.675	primal dual	Byrka et. al.	2015
k -CENTER	2 (tight)	greedy	Hochbaum, Shmoys	1985
OkM	$O(\log n)$	local search	Aouad, Segev	2017
<hr/>				
p -CENTRUM	15	LP rounding	our result	STOC'18
OkM	$38 + \epsilon$	LP rounding	our result	STOC'18
<hr/>				
OkM	$18 + \epsilon$	primal-dual	Chakrabarty et al.	ICALP'18
OkM	$5 + \epsilon$	primal-dual	Chakrabarty et al.	STOC'19

Approximation Algorithms for OkM

Problem	Factor	Technique	Authors	Year
p -CENTRUM	15	LP rounding	our result	STOC'18
OkM	$38 + \epsilon$	LP rounding	our result	STOC'18
OkM	$18 + \epsilon$	primal-dual	Chakrabarty et al.	ICALP'18
OkM	$5 + \epsilon$	primal-dual	Chakrabarty et al.	STOC'19

Open questions:

- better approximation for ORDERED k -MEDIAN
- better approximation for k -FACILITY p -CENTRUM
- better lower bounds
(current: $3 - \epsilon$ hard to approx. unless P=NP)
- consider weighted case (a_i copies of client i)

Main Contributions

- The first polynomial time constant-factor approximation algorithm for the ORDERED k -MEDIAN problem.

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- The first polynomial time constant-factor approximation algorithm for the k -FACILITY p -CENTRUM problem.
- The first polynomial time constant-factor approximation algorithm for the HARMONIC k -MEDIAN problem in general spaces (not only metric).

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- The first PTAS (polynomial time approximation scheme) for the MINIMAX APPROVAL VOTING problem.

- The first polynomial time constant-factor approximation algorithm for the ORDERED k -MEDIAN problem.
- The first polynomial time constant-factor approximation algorithm for the k -FACILITY p -CENTRUM problem.
- The first polynomial time constant-factor approximation algorithm for the HARMONIC k -MEDIAN problem in general spaces (not only metric).
- The first PTAS (polynomial time approximation scheme) for the MINIMAX APPROVAL VOTING problem.
- A parameterized approximation scheme for MINIMAX APPROVAL VOTING parameterized by the value d of an optimal solution. The running time is upperbounded by $(3/\epsilon)^{2d} (nm)^{\mathcal{O}(1)}$. This is essentially optimal assuming the Exponential Time Hypothesis.

- Jarosław Byrka, Krzysztof Sornat and Joachim Spoerhase.
Constant-Factor Approximation for Ordered k-Median.
STOC 2018
- Jarosław Byrka, Piotr Skowron, Krzysztof Sornat.
Proportional Approval Voting, Harmonic k-Median, and Negative Association.
ICALP 2018
- Jarosław Byrka, Krzysztof Sornat.
PTAS for Minimax Approval Voting.
WINE 2014
- Marek Cygan, Łukasz Kowalik, Arkadiusz Socała and Krzysztof Sornat.
Approximation and Parameterized Complexity of Minimax Approval Voting.
AAAI 2017 (preliminary version)
Journal of Artificial Intelligence Research 63, 2018 (full version)

- Jarosław Byrka, Krzysztof Sornat and Joachim Spoerhase.
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Thank you!