Offsets of Rational Surfaces & Ray Tracing.

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Abstract: Offsets or parallel surfaces were used since Leibniz and are important means in robotics, CAGD and tolerance analysis ([2]). Presented algorithm solves the problem of finding all intersections between a line and an offset of a rational parametric surface. The main idea is to eliminate the normal vector from the intersection calculations. The square root which is present in normal vector calculations is responsible for the complicated nature of the offset as well as for the high cost of direct solving of intersection equations. Instead of using the normal vector this algorithm uses extensively the distance function and it is based on Hoffmann dimensionality paradigm [1].

For rational surface $S(u,v)$ of degree (m,n) the intersection problem is reformulated into the problem of solving an array of equations: $F(u,v,t) = 0$, $\partial_u F(u,v,t) = 0$, $\partial_v F(u,v,t) = 0$ where function F is polynomial in Bézier form of degree (2m,2n,2) (F is simplification of $f(u,v,t) = ||S(u,v) - R(t)||^2 - d^2$). Array of equations is solved by subdivision method and the solutions are classified because they can belong either to positive offset $S_d(u,v)$ or to the negative one $S_{-d}(u,v)$.

Simplicity of the algorithm makes it efficient and easy to implement. The complexity analysis and the experimental results show that for polynomial surfaces of degree 2–4 the method is at least 5 times faster than the one proposed by M.E.Vafiadou and N.M.Patrikalakis [3]. If we compare it with the obvious generalization of [3] to rational surfaces our method is at least 30 times faster and it is even faster for the surfaces of higher degree. The algorithm can be used in the ray tracing method for offset visualization as well as in the offset shape interrogation. Example images are presented.

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2. Rossignac, J.R., Requicha, A.A.G., *Offseting operations in solid modelling*, Computer Aided Geometric Design 3 (1986) pp.129–148.
3. Vafiadou, M.E., Patrikalakis, N.M., *Interrogation of Offsets of Polynomial Surface Patches*, Eurographics '91 (Elsevier Science Publishers B.V.) pp.247–259.

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