Compressed Membership for NFA (DFA) with Compressed Labels is in NP (P)

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- (SLP) fully compressed pattern matching (in $\mathcal{O}(n^2)$)

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Results

- Fully compressed membership problem for NFA (in NP)
- Fully compressed membership problem for DFA (in P)
- (SLP) fully compressed pattern matching (in $\mathcal{O}(n^2)$)
- word equations: simple, unified proof for everything that is known

Straight Line Programms SLPs

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Context free grammar defining a single word. (Chomsky normal form).

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SLPs as a compression model

- application (LZ, logarithmic transformation)
- theory (formal languages)
- preserves/captures word properties

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Applied in many proofs and constructions.

Usage and work on SLP

Theory

- word equations (Plandowski: satisfiability in PSPACE)

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- equality
- pattern matching

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Independent interest

- indexing structure for SLP

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- develop tools/gain understanding
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RE, CFG, Conjunctive grammars. . .

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Open questions

- Compressed membership for NFA

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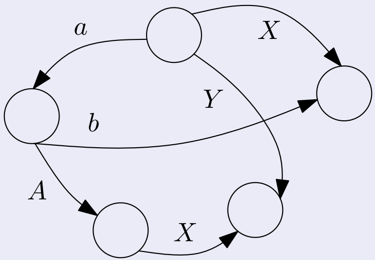

Compressed membership for NFA: complexity

Complexity

- NP-hardness (subsum), already for
  - acyclic NFA
  - unary alphabet
- in PSPACE: enough to store positions inside decompressed words

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Conjecture

In NP.

Partial results

- Plandowski & Rytter (unary in NP)
- Lohrey & Mathissen (highly periodic in NP, highly aperiodic in P)

New results

Theorem

Fully compressed membership for NFA is in NP.

Theorem

Fully compressed membership for DFA is in P.

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Deeper understanding

New production: $d \to ab$. Building new SLP (recompression).
SLP problems: hard, as SLP are different.
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$a_2 \quad c \quad a_3$

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Problems

Easy for text, what about grammar?

Local recompression

Re-compression

- decompressed text: easy; size: large,
- compressed text: hard; size: small.

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Local decompression

Decompress locally the SLP:

$$X \to uYvZ$$

- u, v: blocks of letters, linear size
- Y, Z: nonterminals
- recompression inside u, v

Outline

Outline of the algorithm

while $|\text{val}(X_n) > n|$ **do**

$L_\Sigma \leftarrow$ list of letters, $L_P \leftarrow$ list of pairs

for $ab \in L_P$ **do**

compress pair ab

for $a \in L_\Sigma$ **do**

compress a maximal blocks

Decompress the word and solve the problem naively.

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Theorem

There are $\mathcal{O}(\log|\text{val}(X_n)|)$ *iterations.*

Proof.

Consider two consecutive letters ab. One of them is compressed. So word shortens by a constant factor. □

What is hard, what is easy

What is hard to compress, what easy?

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Hard

- a pair ab is crossing if $X_i \to uaX_jvX_k$, where $\text{val}(X_j) = b \dots$
- a letter a has crossing appearances if aa is a crossing pair

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Easy

- a pair ab is non-crossing otherwise
- a letter a has no crossing appearances otherwise

A little detailed outline

Detailed outline

while $|\text{val}(X_n) > n|$ **do**
  while possible **do**
    for non-crossing pair ab in $\text{val}(X_n)$ **do**
      compress ab
    for a: without crossing blocks **do**
      compress appearances of a

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while $|\text{val}(X_n) > n|$ **do**
  while possible **do**
    for non-crossing pair ab in $\text{val}(X_n)$ **do**
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    for a: without crossing blocks **do**
      compress appearances of a
  $L \leftarrow$ list of letters with crossing blocks
  $P \leftarrow$ list of crossing pairs
  for each ab in P **do**
    compress ab
  for $a \in L$ **do**
    compress appearances of a
Decompress X_n and solve the problem naively.

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Appearance compression for a without crossing blocks

compute the lengths $\ell_1, \ldots, \ell_k$ of a's maximal blocks
for each a^{ℓ_m} **do**
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Lemma

It works.

Proof.

The pair is non-crossing: it always appears inside production. □

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Convert crossing pairs to noncrossing and letters with crossing blocks to letters without crossing blocks (Sequentially).

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Lemma

After popping letters, ab is noncrossing.

Proof.

Easy, some simple cases. □

Removing crossing blocks of a

- aa is a crossing pair: pop a
- can be insufficient
- cut a-prefix or a-suffix
- Represent $\text{val}(X_i)$ as $a^{\ell_i} w a^{r_i}$, turn it into w.

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Changing a letter a with crossing blocks to one without

for $i = 1 \,..\, n$ **do**
  let $X_i \to uX_jvX_k$
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Lemma

After the algorithm a has no crossing block.

Represent a^{ℓ} succinctly, using $\mathcal{O}(\log \ell)$ bits.

Sizes and running time

Running time

All algorithms run in time $poly(n, |G|, |\Sigma|)$.

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$$abbbcceaX_j addfeaaf X_k$$

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$$abbbccea\,bha\,X_j\,ab\,addfeaaf\,cda\,X_k$$

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- $\mathcal{O}(n)$ new letters

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$$uv\,bhaX_j\,ab\,xyz\,cdaX_k$$

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New letters ($|\Sigma|$)

- noncrossing pairs, noncrossing blocks compression (shrinks $|G|$)
- letters with crossing blocks and crossing pairs:
  there are $\mathcal{O}(n)$ such letters and $\mathcal{O}(n^2)$ pairs in $\mathrm{val}(X_n)$

Modifications

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Questions

- Any further results?
- How efficient for DFA?
- Are word equations in NP?