Word Equations: Sheet 8

Task 1 Show how to implement the split operation in the data structure for equality testing of dynamic texts. In particular show that it is enough to modify only $\mathcal{O}(\log^* N)$ elements per level, where N is the bound on the maximal size of the numbers occurring in the signature.

Task 2 Show that $\log^* N = \mathcal{O}(\log^*(\max(m, |\Sigma|)))$, where Σ is the input alphabet and m is the number of performed operations.
Hint[1]

Task 3 Can you apply the signature-building algorithm from the data structure for equality of dynamic texts to word equations? What assumptions do you need? What is the size of the equation?

Task 4 Prove the basic lemma: All occurrences of $\text{val}(\mathcal{P})$ in $\text{val}(\mathcal{T})$ overlapping any given position form a single arithmetical progression.

Task 5 Give a procedure for intersecting two arithmetic progressions represented as triples (*first, step, end*), i.e. the first element, the step of the arithmetic progression and the last element.

Task 6 Show that the recursion in the $LSP(i, j, [\alpha, \beta])$ has at most 3 active branches (so one that do not terminate immediately). Deduce from this that the running time of $LSP(i, j, [\alpha, \beta])$ indeed takes $\mathcal{O}(j)$ time.

Task 7 Show that the result of the Local Search Procedure is always a collection of at most two arithmetic progressions.
Hint[2]

[1] You can use the bound calculated in the previous task, assuming that it also applies to concatenation.
[2] Basic Lemma